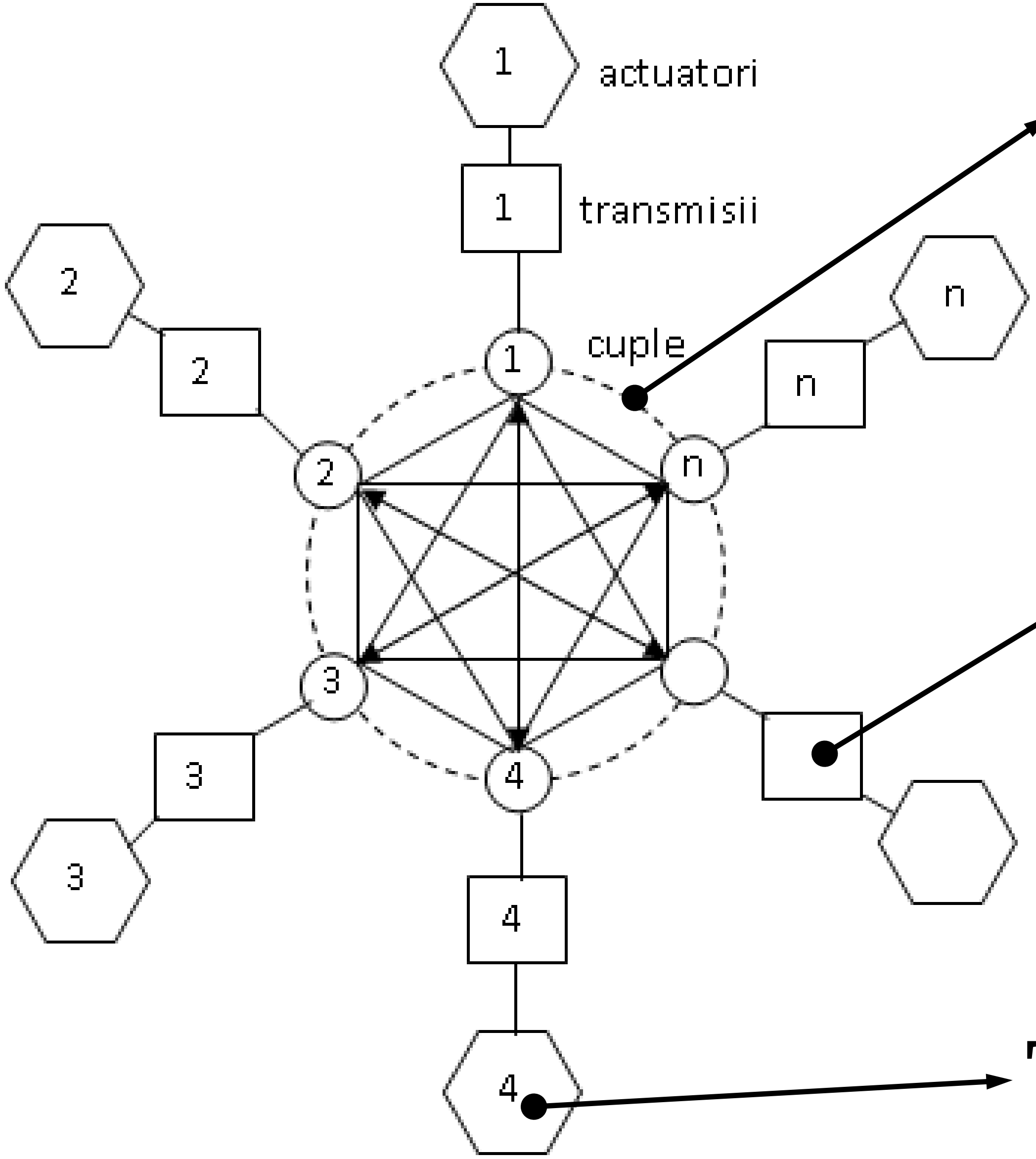
The Manipulators models_4

The Dynamic Model

Agenda

- **The Dynamical model construction**
- **The Dynamical model of the Mechanical structure**
- **The Dynamical model of the Mechanical transmission**
- **The Dynamical model of the DC Motors**
- **The Manipulator dynamical model**

THE DYNAMICAL MODEL CONSTRUCTION



$$\mathbf{f_s(q,\dot{q},\ddot{q}) = T}$$

$$\mathbf{T(\dot{q}) = T_a(\dot{q}_a)}$$

$$\mathbf{T_a(q_a,\dot{q}_a,\ddot{q}_a) = V}$$

$$\mathbf{f(q,\dot{q},\ddot{q}) = V}$$

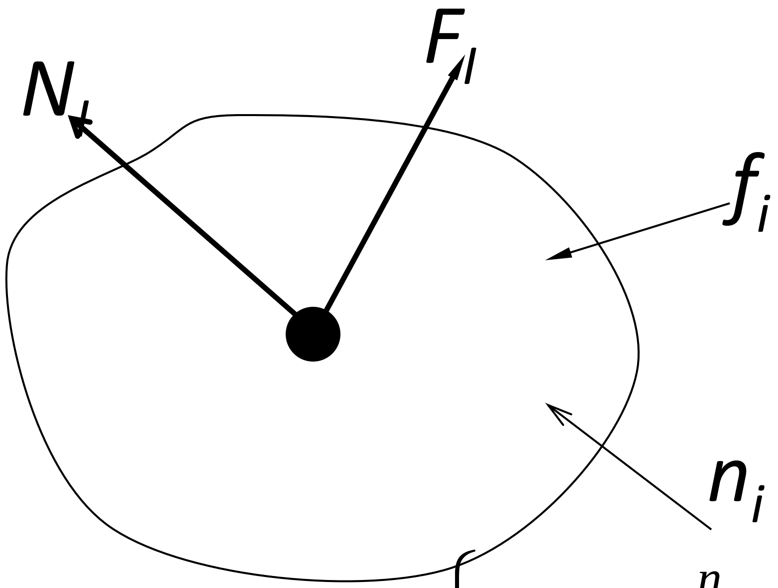
THE MECHANICAL STRUCTURE DYNAMICAL MODEL

$$\mathbf{f_s}(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}}) = \mathbf{T}$$

FM

$$\mathbf{M}(\mathbf{q})\ddot{\mathbf{q}} + \mathbf{V}(\mathbf{q}, \dot{\mathbf{q}}) + \mathbf{F_f}(\dot{\mathbf{q}}) + \mathbf{G}(\mathbf{q}) + \mathbf{J}(\mathbf{q})^{\mathrm{T}}\mathbf{F} = \mathbf{T}$$

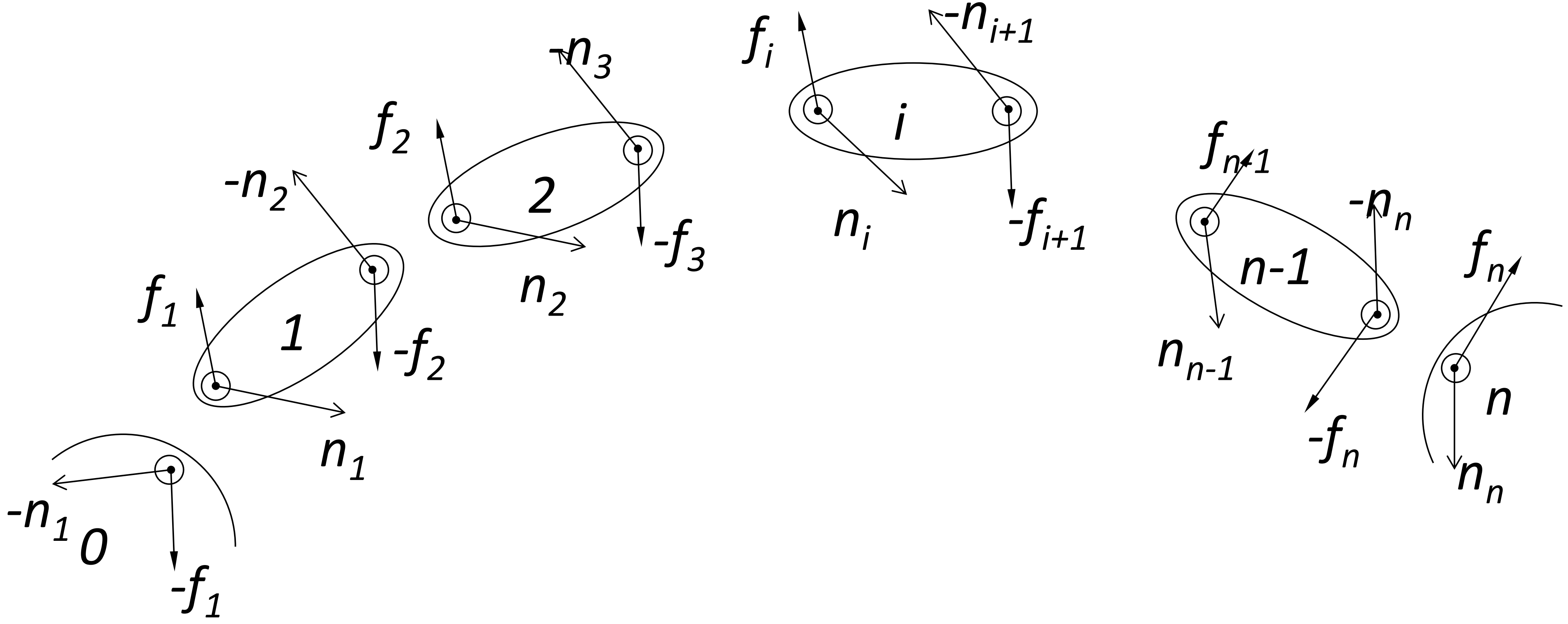
NE



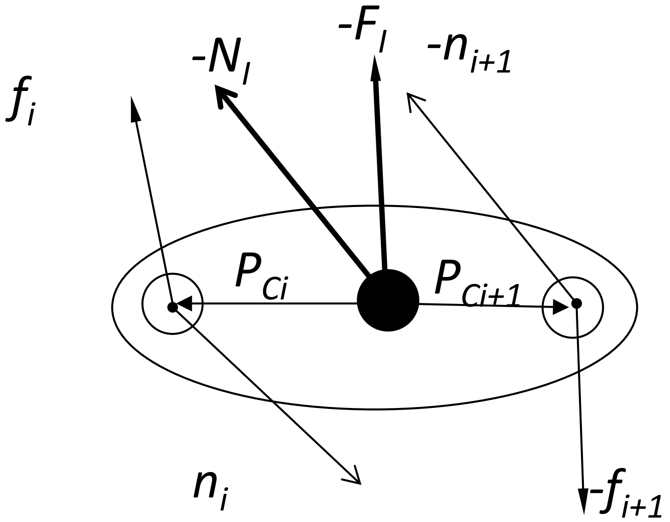
$$\begin{Bmatrix} \sum_{i=1}^{n} \mathbf{f}_i \\ \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{f}_i + \sum_{i=1}^{n} \mathbf{n}_i \end{Bmatrix} = \begin{Bmatrix} \mathbf{F}_\mathbf{I} \\ \mathbf{N}_\mathbf{I} \end{Bmatrix}$$

$$\mathbf{F}_\mathbf{I} = m\ddot{\mathbf{x}}$$

$$\mathbf{N}_\mathbf{I} = \mathbf{I}_\mathbf{C}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I}_\mathbf{C}\boldsymbol{\omega}$$



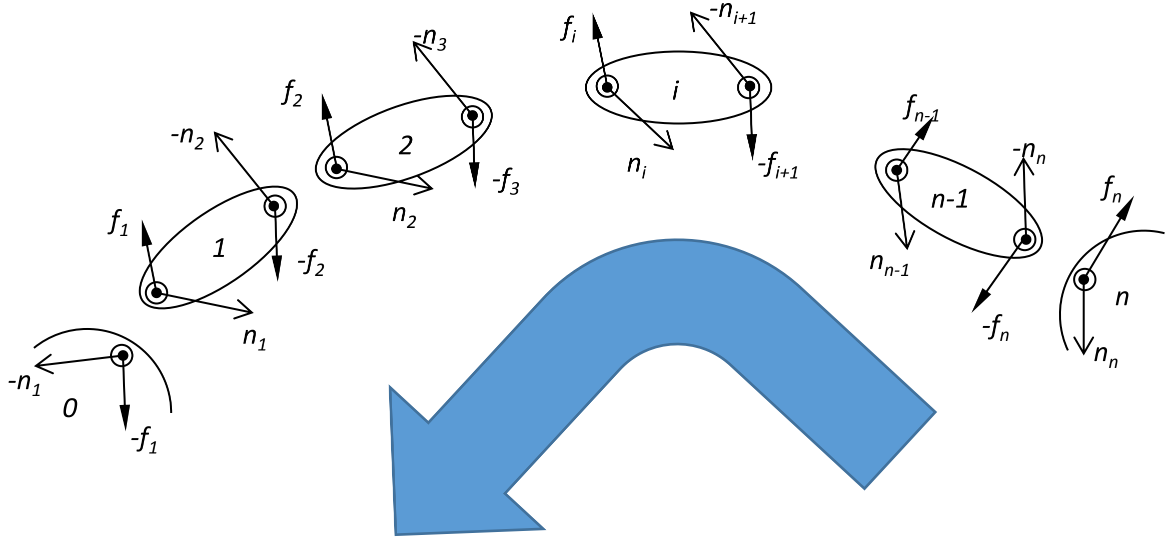
0
-n1
-f1
1
f1
n1
-n2
-f2
2
f2
n2
-n3
-f3
i
fi
ni
-ni+1
-fi+1
n-1
fn-1
nn-1
-nn
-fn
n
fn
nn



$$\tau_i = \begin{cases} \mathbf{n_i} \cdot \mathbf{Z_i}; rotatie \\ \mathbf{f_i} \cdot \mathbf{Z_i}; translatie \end{cases}$$

$$\begin{Bmatrix} \sum_{i=1}^{n} \mathbf{f}_i \\ \sum_{i=1}^{n} \mathbf{r}_i \times \mathbf{f}_i + \sum_{i=1}^{n} \mathbf{n}_i \end{Bmatrix} = \begin{Bmatrix} \mathbf{F_I} \\ \mathbf{N_I} \end{Bmatrix}$$

$$\begin{cases} \mathbf{F_I} = \mathbf{f_i} - \mathbf{f_{i+1}} \\ \mathbf{N_I} = \mathbf{n_i} - \mathbf{n_{i+1}} + \mathbf{p_{Ci}} \times \mathbf{f_i} - \mathbf{p_{Ci+1}} \times \mathbf{f_{i+1}} \end{cases}$$



f_i
$-n_{i+1}$
$-n_3$
f_2
i
$-n_2$
2
f_{n-1}
$-f_3$
n_i
$-f_{i+1}$
$-n_n$
$n-1$
f_n
f_1
n_2
1
$-f_2$
n_{n-1}
n
n_1
$-f_n$
$-n_1$
n_n
0
$-f_1$

$$\frac{d(m\mathbf{v})}{dt} = \mathbf{F}$$

Impulsul
$\mathbf{H} = m\mathbf{v}$
Ecuația lui Newton
$\dot{\mathbf{H}} = m\mathbf{a} = \mathbf{F}_\mathbf{I}$

Momentul cinetic

$$\mathbf{K} = \mathbf{I_O}\boldsymbol{\omega}$$

Ecuația lui Euler

$$\dot{\mathbf{K}} = \mathbf{I_O}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I_O}\boldsymbol{\omega} = \mathbf{N_I}$$

$$\frac{d(\mathbf{r} \times m\mathbf{v})}{dt} = \mathbf{N}$$

$$\mathbf{K} = \int_D \mathbf{r} \times \mathbf{v}\,dm = \int_D \mathbf{r} \times (\boldsymbol{\omega} \times \mathbf{r})\,dm = \int_D \tilde{\mathbf{r}}(-\tilde{\mathbf{r}})\boldsymbol{\omega}\,dm = \left(\int_D \tilde{\mathbf{r}}(-\tilde{\mathbf{r}})\,dm\right)\boldsymbol{\omega} \qquad \mathbf{K} = \mathbf{I_O}\boldsymbol{\omega}$$

$$\mathbf{I_O} = \int_D \tilde{\mathbf{r}}(-\tilde{\mathbf{r}})\rho\,dV = \int_D \begin{bmatrix} 0 & -z & y \\ z & 0 & -x \\ -y & x & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & z & -y \\ -z & 0 & x \\ y & -x & 0 \end{bmatrix} \rho\,dV = \int_D \begin{bmatrix} y^2+z^2 & -xy & -xz \\ -xy & x^2+z^2 & -zy \\ -xz & -zy & y^2+x^2 \end{bmatrix} \rho\,dV$$

$$\mathbf{I_O} = \begin{bmatrix} I_{xx} & -I_{xy} & -I_{xz} \\ -I_{xy} & I_{yy} & -I_{yz} \\ -I_{xz} & -I_{yz} & I_{zz} \end{bmatrix}$$

$$\dot{\mathbf{K}} = \mathbf{I_O}\dot{\boldsymbol{\omega}} + \boldsymbol{\omega} \times \mathbf{I_O}\boldsymbol{\omega} = \mathbf{N_I}$$

$$\boldsymbol{\omega}_{\mathbf{i+1}} = \boldsymbol{\omega}_{\mathbf{i}} + \dot{\theta}_{i+1}\mathbf{Z}_{\mathbf{i+1}}$$

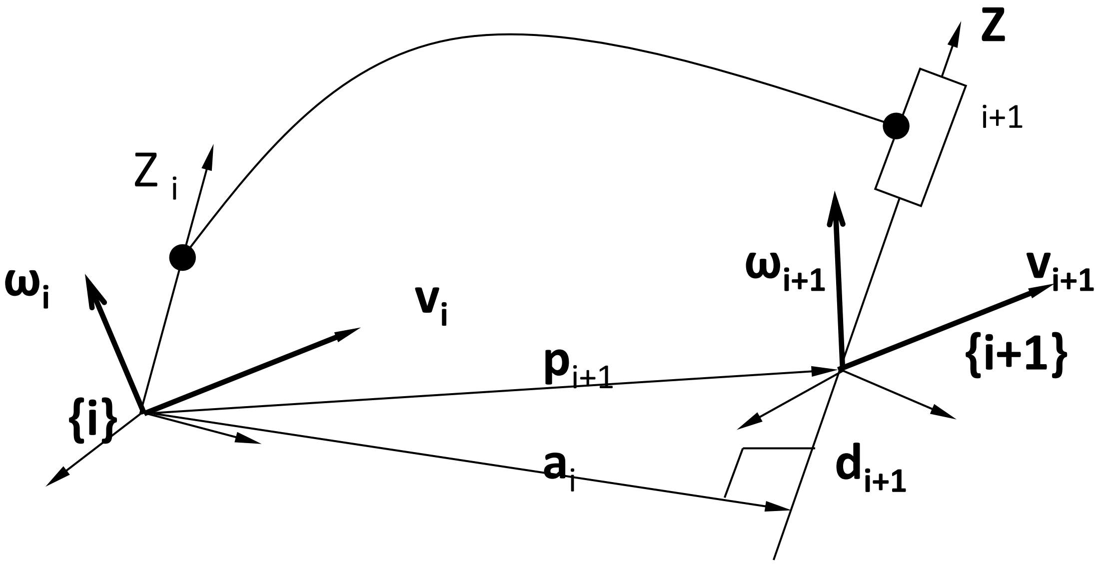
$$\dot{\boldsymbol{\omega}}_{\mathbf{i+1}} = \dot{\boldsymbol{\omega}}_{\mathbf{i}} + \dot{\theta}_{i+1}(\boldsymbol{\omega}_{\mathbf{i}} \times \mathbf{Z}_{\mathbf{i+1}}) + \ddot{\theta}_{i+1}\mathbf{Z}_{\mathbf{i+1}}$$

$$\dot{\mathbf{H}} = m\mathbf{a} = \mathbf{F_I}$$

$$\mathbf{v_{i+1}} = \mathbf{v_i} + \boldsymbol{\omega}_\mathbf{i} \times \mathbf{p_{i+1}} + \dot{d}_{i+1}\mathbf{Z_{i+1}}$$

$$\dot{\mathbf{v}}_\mathbf{i+1} = \dot{\mathbf{v}}_\mathbf{i} + \dot{\boldsymbol{\omega}}_\mathbf{i} \times \mathbf{p_{i+1}} + \dot{\boldsymbol{\omega}}_\mathbf{i} \times \dot{\mathbf{p}}_\mathbf{i+1} + \boldsymbol{\omega}_\mathbf{i} \times (\boldsymbol{\omega}_\mathbf{i} \times \mathbf{p_{i+1}}) + \ddot{d}_{i+1}\mathbf{Z_{i+1}} + \boldsymbol{\omega}_\mathbf{i} \times \dot{d}_{i+1}\mathbf{Z_{i+1}}$$

$$\dot{\mathbf{v}}_\mathbf{i+1} = \dot{\mathbf{v}}_\mathbf{i} + \dot{\boldsymbol{\omega}}_\mathbf{i} \times \mathbf{p_{i+1}} + \boldsymbol{\omega}_\mathbf{i} \times (\boldsymbol{\omega}_\mathbf{i} \times \mathbf{p_{i+1}}) + \ddot{d}_{i+1}\mathbf{Z_{i+1}} + 2\dot{d}_{i+1}\boldsymbol{\omega}_\mathbf{i} \times \mathbf{Z_{i+1}}$$

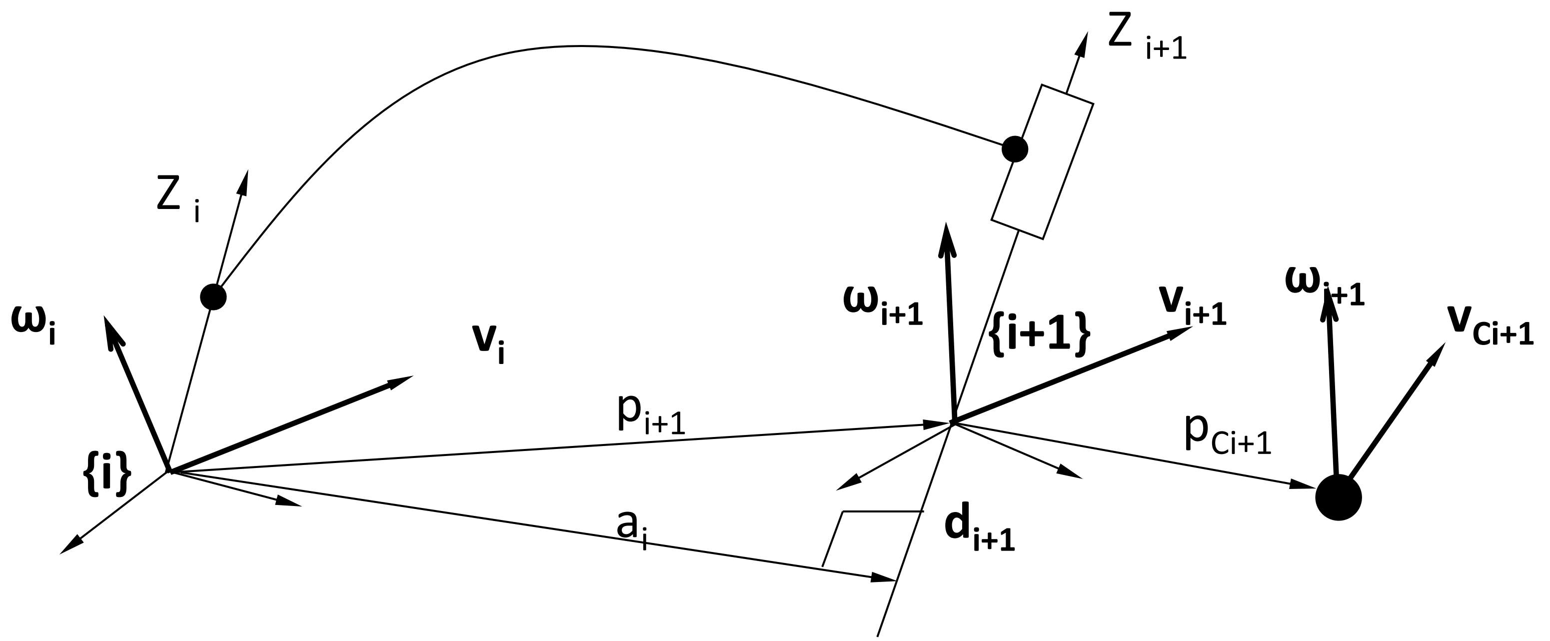


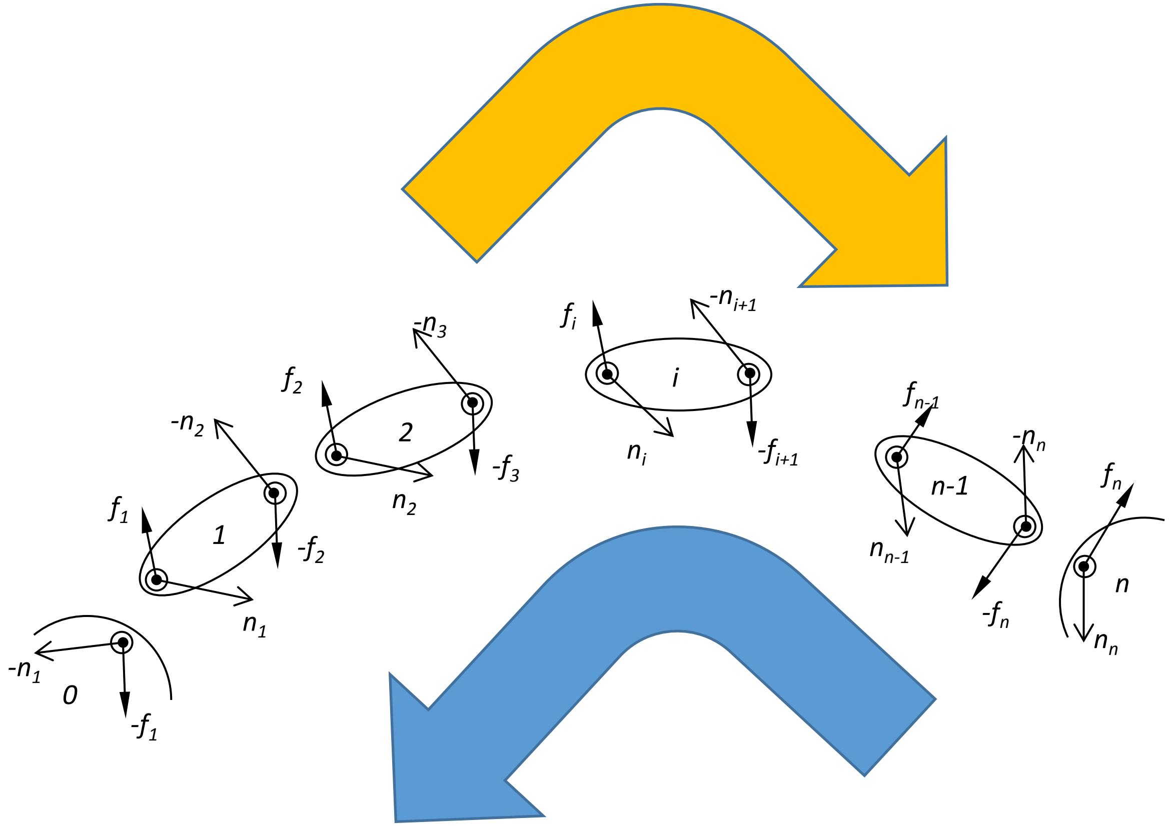
$$\boldsymbol{p}_{i+1} = \boldsymbol{a}_i + d_{i+1}\boldsymbol{Z}_{i+1}$$

$$\dot{\mathbf{H}} = m\mathbf{a} = \mathbf{F_I}$$

$$\mathbf{v_{Ci+1}} = \mathbf{v_{i+1}} + \boldsymbol{\omega}_{\mathbf{i+1}} \times \mathbf{p_{Ci+1}}$$

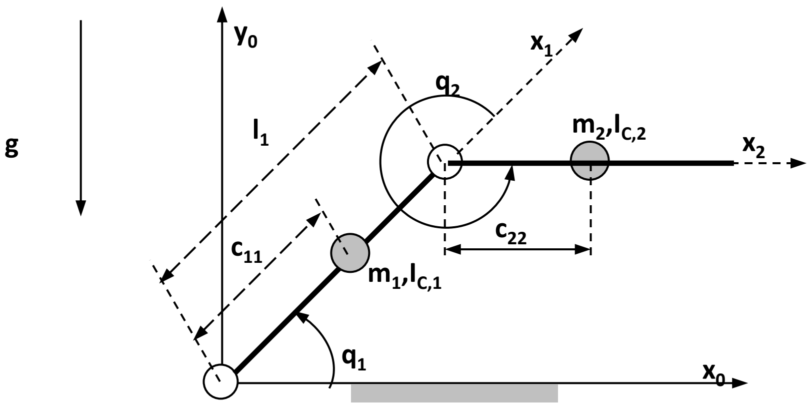
$$\dot{\mathbf{v}}_{\mathbf{Ci+1}} = \dot{\mathbf{v}}_{\mathbf{i+1}} + \dot{\boldsymbol{\omega}}_{\mathbf{i+1}} \times \mathbf{p_{Ci+1}} + \boldsymbol{\omega}_{\mathbf{i+1}} \times (\boldsymbol{\omega}_{\mathbf{i+1}} \times \mathbf{p_{Ci+1}})$$





$-n_{i+1}$
f_i
$-n_3$
i
f_2
2
f_{n-1}
$-n_2$
$-n_n$
$-f_3$
n_i
$-f_{i+1}$
f_n
$n-1$
n_2
f_1
1
$-f_2$
n_{n-1}
n
n_1
$-f_n$
n_n
$-n_1$
0
$-f_1$

NE_v



$$^0\mathbf{p}_1 = \begin{bmatrix} c_{11}cq_1 \\ c_{11}sq_1 \\ 0 \end{bmatrix} \qquad ^0\ddot{\mathbf{p}}_1 = \begin{bmatrix} -c_{11}cq_1\dot{q}_1^{\,2} - c_{11}sq_1\ddot{q}_1 \\ -c_{11}sq_1\dot{q}_1^2 + c_{11}cq_1\ddot{q}_1 \\ 0 \end{bmatrix}$$

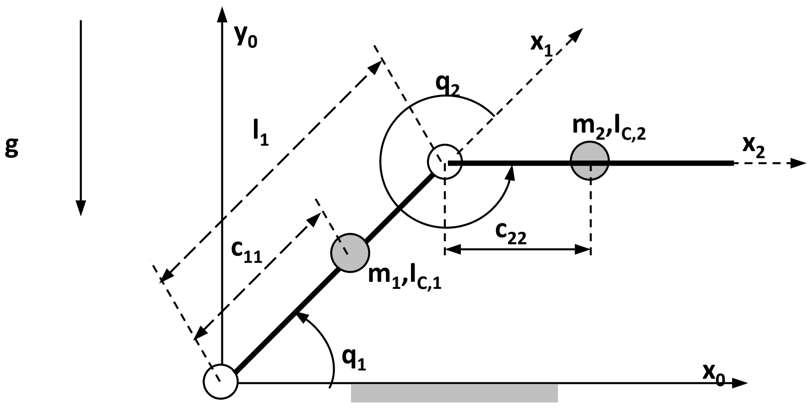
$$^0\mathbf{p}_2 = \begin{bmatrix} l_1cq_1 + c_{22}cq_{12} \\ l_1sq_1 + c_{22}sq_{12} \\ 0 \end{bmatrix} \qquad ^0\ddot{\mathbf{p}}_2 = \begin{bmatrix} -l_1cq_1\dot{q}_1^2 - l_1sq_1\ddot{q}_1 - c_{22}cq_{12}\dot{q}_{12}^2 - c_{22}sq_{12}\ddot{q}_{12} \\ -l_1sq_1\dot{q}_1^2 + l_1cq_1\ddot{q}_1 - c_{22}sq_{12}\dot{q}_{12}^2 + c_{22}cq_{12}\ddot{q}_{12} \\ 0 \end{bmatrix}$$

$$^1\ddot{\mathbf{p}}_1 = {}^1_0\mathbf{R}\,{}^0\ddot{\mathbf{p}}_1 = \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot {}^0\ddot{\mathbf{p}}_1 = \begin{bmatrix} -c_{11}\dot{q}_1^{\,2} \\ c_{11}\ddot{q}_1 \\ 0 \end{bmatrix}$$

$$^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} \qquad ^1\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix}$$

$$^2\ddot{\mathbf{p}}_2 = {}^2_0\mathbf{R}\,{}^0\ddot{\mathbf{p}}_2 = \begin{bmatrix} cq_{12} & sq_{12} & 0 \\ -sq_{12} & cq_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot {}^0\ddot{\mathbf{p}}_2 = \begin{bmatrix} -l_1cq_2\dot{q}_1^{\,2} + l_1sq_2\ddot{q}_1 - c_{22}\dot{q}_{12}^{1} \\ l_1sq_2\dot{q}_1^{\,2} + l_1cq_2\ddot{q}_1 + c_{22}\ddot{q}_{12} \\ 0 \end{bmatrix}$$

$$^2\omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_{12} \end{bmatrix} \qquad ^1\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_{12} \end{bmatrix}$$



$$
{}^1\ddot{\mathbf{p}}_1 = {}^1_0\mathbf{R}\,{}^0\ddot{\mathbf{p}}_1 = \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot {}^0\ddot{\mathbf{p}}_1 = \begin{bmatrix} -c_{11}\dot{q}_1^2 \\ c_{11}\ddot{q}_1 \\ 0 \end{bmatrix}
$$

$$
{}^1\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} \qquad {}^1\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix}
$$

$$
{}^1\ddot{\mathbf{p}}_1 = {}^1_0\mathbf{R}\,{}^0\ddot{\mathbf{p}}_1 = \begin{bmatrix} cq_{12} & sq_{12} & 0 \\ -sq_{12} & cq_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot {}^0\ddot{\mathbf{p}}_2 = \begin{bmatrix} -l_1cq_2\dot{q}_1^2 + l_1sq_2\ddot{q}_1 - c_{22}\dot{q}_{12}^1 \\ l_1sq_2\dot{q}_1^2 + l_1cq_2\ddot{q}_1 + c_{22}\ddot{q}_{12} \\ 0 \end{bmatrix}
$$

$$
{}^2\omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_{12} \end{bmatrix} \qquad {}^1\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_{12} \end{bmatrix}
$$

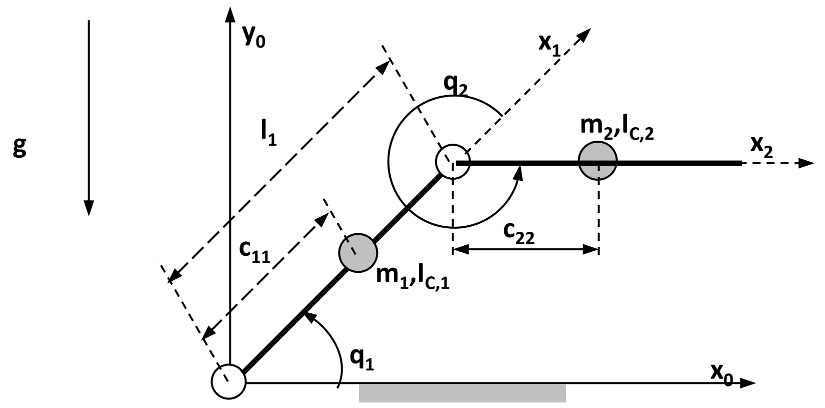
$$
\mathbf{f}_2 = m_2\,{}^2\ddot{\mathbf{p}}_2
$$

$$
\mathbf{n}_2 = \mathbf{I}_{C2}\,{}^2\dot{\omega}_2 + {}^2\tilde{\omega}_2\mathbf{I}_{C2}\,{}^2\omega_2 - \tilde{c}_{22}\mathbf{f}_2
$$

$$
\mathbf{n}_2 = \begin{bmatrix} I_{xx,2} & 0 & 0 \\ 0 & I_{yy,2} & 0 \\ 0 & 0 & I_{zz,2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_{12} \end{bmatrix} + \begin{bmatrix} 0 & -\dot{q}_{12} & 0 \\ \dot{q}_{12} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{xx,2} & 0 & 0 \\ 0 & I_{yy,2} & 0 \\ 0 & 0 & I_{zz,2} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \dot{q}_{12} \end{bmatrix}
$$

$$
- m_2 \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & c_{22} \\ 0 & -c_{22} & 0 \end{bmatrix} \cdot \begin{bmatrix} -l_1cq_2\dot{q}_1^2 + l_1sq_2\ddot{q}_1 - c_{22}\dot{q}_{12}^1 \\ l_1sq_2\dot{q}_1^2 + l_1cq_2\ddot{q}_1 + c_{22}\ddot{q}_{12} \\ 0 \end{bmatrix} =
$$

$$
= \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1(m_2c_{22}(l_1cq_1 + c_{22}) + I_{zz,2}) + \ddot{q}_2(m_2c_{22} + I_{zz,2}) + m_2l_1c_{22}sq_2\dot{q}_1 \end{bmatrix}
$$



$$^{1}\ddot{\mathbf{p}}_1 = {}^{1}_{0}\mathbf{R}\,{}^{0}\ddot{\mathbf{p}}_1 = \begin{bmatrix} cq_1 & sq_1 & 0 \\ -sq_1 & cq_1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot {}^{0}\ddot{\mathbf{p}}_1 = \begin{bmatrix} -c_{11}\dot{q}_1^2 \\ c_{11}\ddot{q}_1 \\ 0 \end{bmatrix}$$

$$^{1}\omega_1 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix} \quad {}^{1}\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix}$$

$$^{1}\ddot{\mathbf{p}}_1 = {}^{1}_{0}\mathbf{R}\,{}^{0}\ddot{\mathbf{p}}_1 = \begin{bmatrix} cq_{12} & sq_{12} & 0 \\ -sq_{12} & cq_{12} & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot {}^{0}\ddot{\mathbf{p}}_2 = \begin{bmatrix} -l_1 cq_2\dot{q}_1^2 + l_1 sq_2\ddot{q}_1 - c_{22}\dot{q}_{12}^1 \\ l_1 sq_2\dot{q}_1^2 + l_1 cq_2\ddot{q}_1 + c_{22}\ddot{q}_{12} \\ 0 \end{bmatrix}$$

$$^{2}\omega_2 = \begin{bmatrix} 0 \\ 0 \\ \dot{q}_{12} \end{bmatrix} \quad {}^{1}\dot{\omega}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_{12} \end{bmatrix}$$

$$\mathbf{f}_1 = m_2\,{}^{1}\ddot{\mathbf{p}}_1 - {}^{1}\mathbf{f}_2$$

$$^{1}\mathbf{n}_1 = \mathbf{I}_{C1}\,{}^{1}\dot{\omega}_1 + {}^{1}\tilde{\omega}_1\mathbf{I}_{C1}\,{}^{1}\omega_1 - \tilde{c}_{12}\,{}^{1}\mathbf{f}_2 - \tilde{c}_{11}\,{}^{1}\mathbf{f}_1 - {}^{1}\mathbf{n}_2$$

$$^{1}\mathbf{f}_2 = -{}^{1}_{2}\mathbf{R}\,{}^{2}\mathbf{f}_2 = \begin{bmatrix} cq_2 & -sq_2 & 0 \\ sq_2 & cq_2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot {}^{2}\mathbf{f}_2 = -m_2 \cdot \begin{bmatrix} -l_1\dot{q}_1^2 - c_{22}cq_2\ddot{q}_1 - c_{22}sq_2\ddot{q}_{12} \\ l_1\ddot{q}_1 + c_{22}cq_2\ddot{q}_{12} - c_{22}sq_2\dot{q}_{12}^2 \\ 0 \end{bmatrix}$$

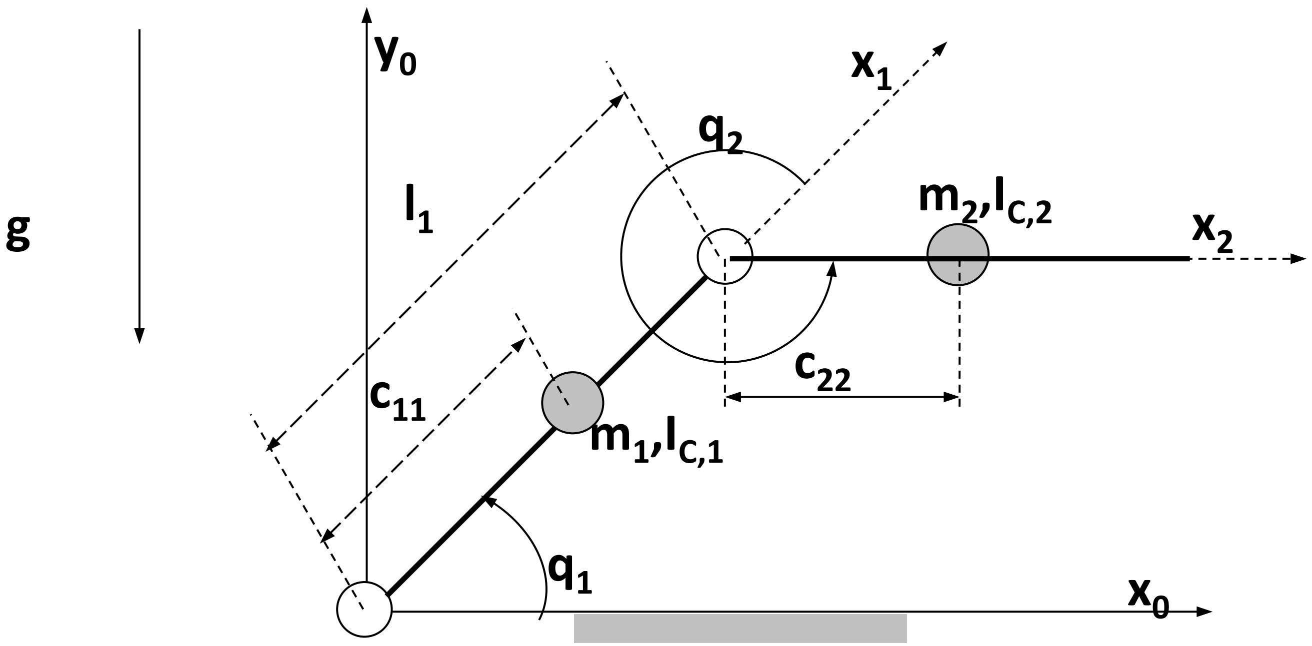
$$^{1}\mathbf{n}_2 = -{}^{1}_{2}\mathbf{R}\,{}^{2}\mathbf{n}_2 = -{}^{2}\mathbf{n}_2$$

$$^{1}\mathbf{n}_1 = \begin{bmatrix} I_{xx,1} & 0 & 0 \\ 0 & I_{yy,1} & 0 \\ 0 & 0 & I_{zz},1 \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & c_{11} \\ 0 & -c_{11} & 0 \end{bmatrix} \cdot \begin{bmatrix} I_{xx,1} & 0 & 0 \\ 0 & I_{yy,1} & 0 \\ 0 & 0 & I_{zz,1} \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 0 \\ \dot{q}_1 \end{bmatrix}$$

$$- \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -(l_1 - c_{11}) \\ 0 & l_1 - c_{11} & 0 \end{bmatrix} \cdot {}^{1}\mathbf{f}_2 - \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & c_{11} \\ 0 & -c_{11} & 0 \end{bmatrix} \cdot X\,{}^{1}\mathbf{f}_1 - {}^{2}\mathbf{n}_2$$

$$^{1}\mathbf{n}_1 = \begin{bmatrix} 0 \\ 0 \\ \ddot{q}_1(m_1c_{11} + m_2(l_1^2 + c_{22}^2) + 2m_2l_1c_{22}cq_2 + I_{zz,1} + I_{zz,2}) + \ddot{q}(m_2c_{22}^2 + m_2l_1c_{22}cq_2 + I_{zz,2}) \end{bmatrix}$$

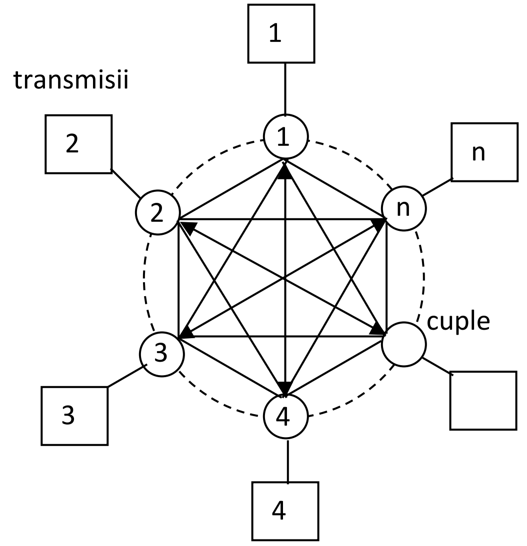
$$+ \begin{bmatrix} 0 \\ 0 \\ -m_2l_1c_{22}sq_2 - 2m_2c_{22}l_1sq_2\dot{q}_1\dot{q}_2 \end{bmatrix}$$



$$\begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}=\begin{bmatrix} m_1c_{11}^2+m_2\left(l_1^2+c_{22}^2\right)+2m_2l_1c_{22}cq_2+{}^1I_{zz,1}+{}^2I_{zz,2} & m_2c_{22}^2+m_2l_1c_{22}cq_2+I_{zz,2}\\ m_2c_{22}^2+m_2l_1c_{22}cq_2+I_{zz,2} & m_2c_{22}+I_{zz,2}\end{bmatrix}\cdot\begin{bmatrix}\ddot{q}_1\\ \ddot{q}_2\end{bmatrix}+$$

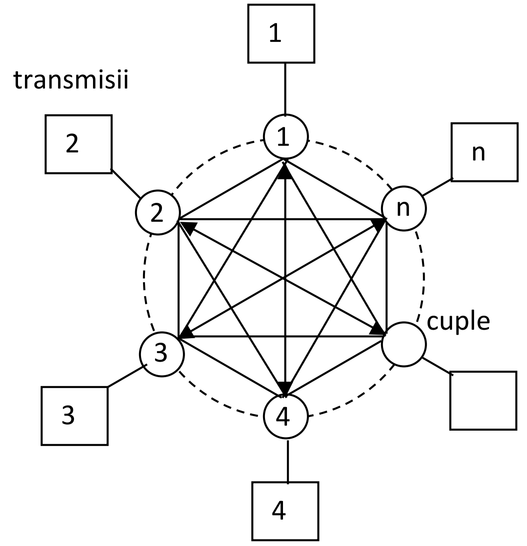
$$+\begin{bmatrix}-m_2l_1c_{22}sq_2\dot{q}_2^2-2m_2c_{22}l_1sq_2\dot{q}_1\dot{q}_2\\ m_2l_1c_{22}sq_2\dot{q}_1^2\end{bmatrix}$$

THE DYNAMICAL MODEL OF THE MECHANICAL TRANSMITION



$$\mathbf{T}(\dot{\mathbf{q}}) = \mathbf{T_a}(\dot{\mathbf{q}}_\mathbf{a})$$

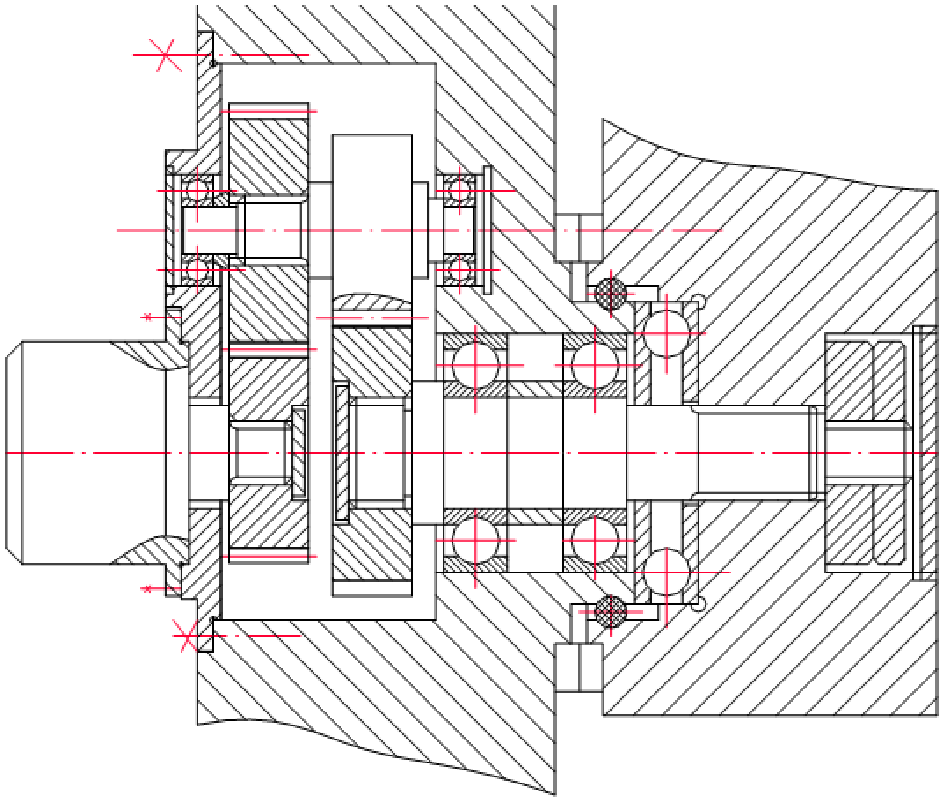
$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \dots \\ \tau_n \end{bmatrix} = \begin{bmatrix} i_{R1} & 0 & . & 0 \\ 0 & i_{R2} & . & 0 \\ . & . & . & 0 \\ 0 & 0 & . & i_{Rn} \end{bmatrix} \cdot \begin{bmatrix} \tau_{a,1} \\ \tau_{a,2} \\ . \\ \tau_{a,n} \end{bmatrix}$$

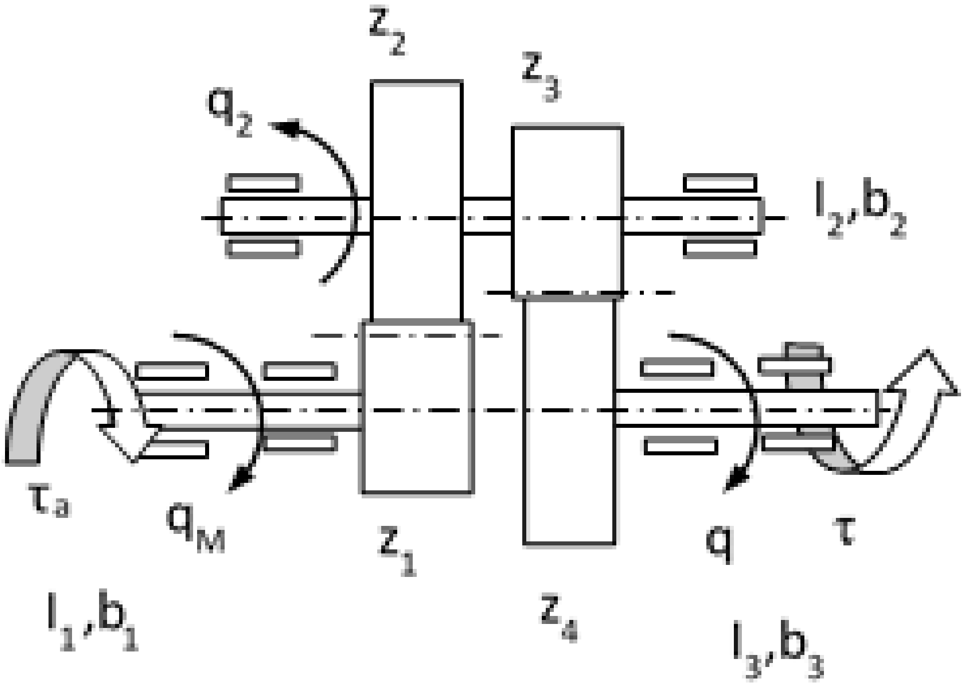


$$\mathbf{T}(\dot{\mathbf{q}}) = \mathbf{T_a}(\dot{\mathbf{q}}_\mathbf{a})$$

$$\begin{bmatrix} \tau_1 \\ \tau_2 \\ \ldots \\ \tau_n \end{bmatrix} = \begin{bmatrix} i_{R1} & 0 & . & 0 \\ 0 & i_{R2} & . & 0 \\ . & . & . & 0 \\ 0 & 0 & . & i_{Rn} \end{bmatrix} . \begin{bmatrix} \tau_{a,1} \\ \tau_{a,2} \\ . \\ \tau_{a,n} \end{bmatrix} - \begin{bmatrix} I_{ech1} & 0 & . & 0 \\ 0 & I_{ech2} & . & 0 \\ . & . & . & 0 \\ 0 & 0 & . & I_{echn} \end{bmatrix} . \begin{bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ . \\ \ddot{q}_n \end{bmatrix} - \begin{bmatrix} b_{ech1} & 0 & . & 0 \\ 0 & b_{ech2} & . & 0 \\ . & . & . & 0 \\ 0 & 0 & . & b_{echn} \end{bmatrix} . \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ . \\ \dot{q}_n \end{bmatrix}$$

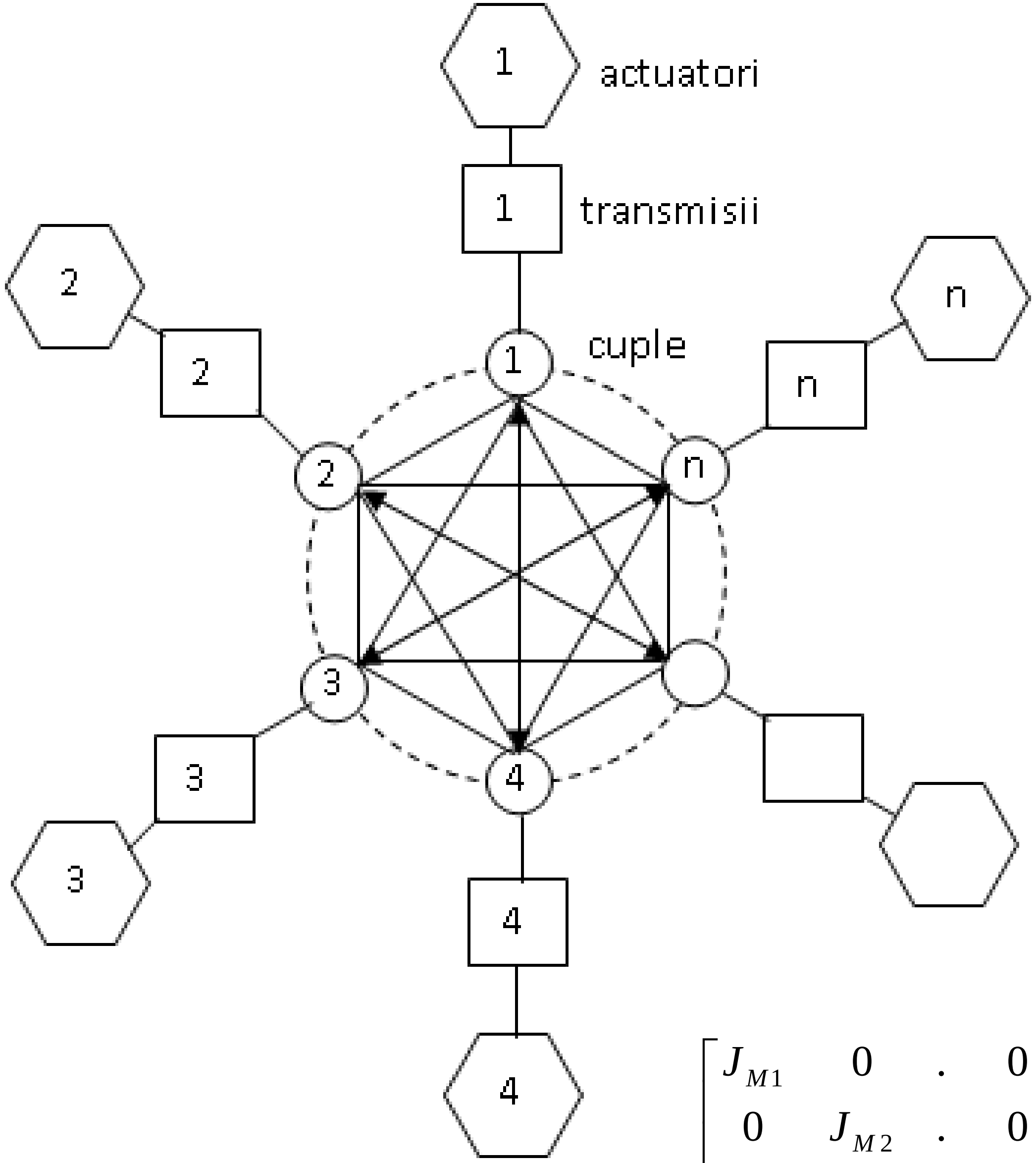
MT_e





$$\begin{cases} \tau = \tau_a \dfrac{\omega_M}{\omega} \\ \dfrac{\omega_M}{\omega} = i_R \end{cases} \Rightarrow \tau = \tau_a \cdot i_R$$

THE DC MOTORS DYNAMICAL MODEL

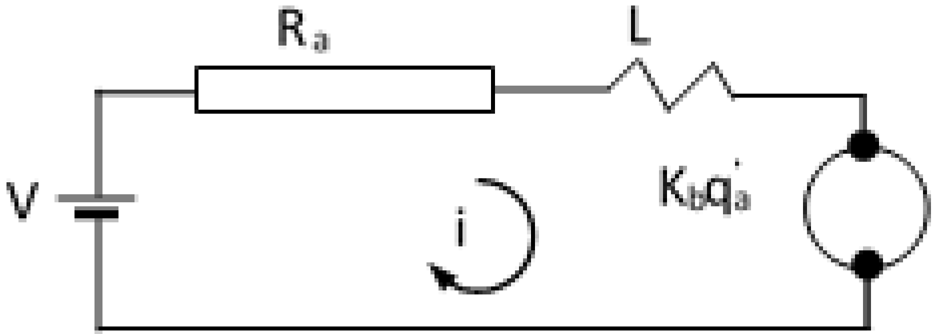


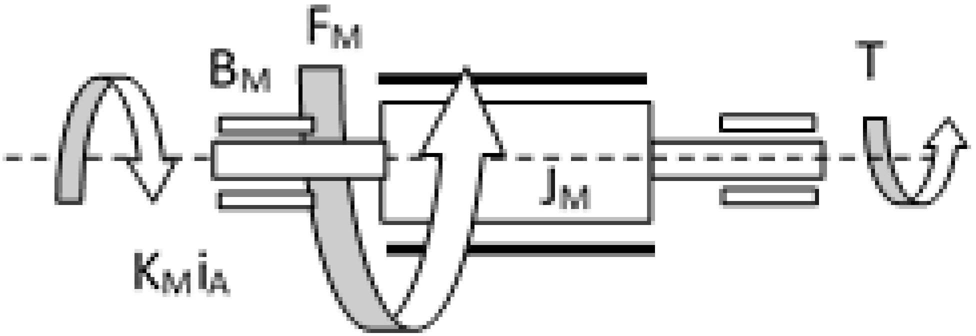
MA

$$\mathbf{T_a(q_a,\dot{q}_a,\ddot{q}_a)=V}$$

$$\begin{bmatrix} J_{M1} & 0 & . & 0 \\ 0 & J_{M2} & . & 0 \\ . & . & . & . \\ 0 & 0 & . & J_{Mn} \end{bmatrix} \cdot \begin{bmatrix} \ddot{q}_{a1} \\ \ddot{q}_{a2} \\ . \\ q_{an} \end{bmatrix} + \begin{bmatrix} B_1 & 0 & . & 0 \\ 0 & B_2 & . & 0 \\ . & . & . & . \\ 0 & 0 & . & B_n \end{bmatrix} . \cdot \begin{bmatrix} \dot{q}_{a1} \\ \dot{q}_{a2} \\ . \\ \dot{q}_{an} \end{bmatrix} + \begin{bmatrix} F_{M1} \\ F_{M2} \\ . \\ F_{Mn} \end{bmatrix} + \begin{bmatrix} T_1 \\ T_2 \\ . \\ T_n \end{bmatrix} = \begin{bmatrix} \frac{K_{M1}}{R_{a1}} & 0 & . & 0 \\ 0 & \frac{K_{M2}}{R_{a2}} & . & 0 \\ . & . & . & \\ 0 & 0 & . & \frac{K_{Mn}}{R_{an}} \end{bmatrix} \cdot \begin{bmatrix} V_1 \\ V_2 \\ . \\ V_n \end{bmatrix}$$

MA_e





$$\begin{cases} R_a i = V - K_b \dot{q}_a \\ \tau_a = K_M \cdot i \\ J_M \ddot{q}_a = \tau_M - B_M \dot{q}_a - T - F_M \end{cases} \Rightarrow \begin{cases} \tau_a = \dfrac{K_M}{R_a} \cdot V - \dfrac{K_b K_M}{R_a} \dot{q}_a \\ J_M \ddot{q}_M = \tau_M - B_M \dot{q}_a - T - F_M \end{cases}$$

$$\Rightarrow \quad J_M \ddot{q}_a + \left(\frac{K_b K_M}{R_a} + B_M\right) \cdot \dot{q}_a + T + F_M = \frac{K_M}{R_a} V$$

THE MANIPULATOR DYNAMICAL MODEL

$$
\begin{cases}
\mathbf{M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F_f(\dot{q}) + G(q) + J^T(q)F + T_P = T} \\
\mathbf{J}_{ech}\mathbf{\ddot{q}} + \mathbf{b}_{ech}\mathbf{\dot{q}} + \mathbf{T} = \mathbf{i_R T_a} \\
\mathbf{J_M \ddot{q}_a + B\dot{q}_a + F_M + T_a = K_M V} \\
\mathbf{q_a = i_R^{-1} q}
\end{cases}
$$

$$
\begin{cases}
\mathbf{M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F_f(\dot{q}) + G(q) + J^T(q)F + T_P = T} \\
\mathbf{J}_{ech}\mathbf{\ddot{q}} + \mathbf{b}_{ech}\mathbf{\dot{q}} + \mathbf{T} = \mathbf{i_R T_a} \\
\mathbf{J_M i_R^{-1} \ddot{q} + B i_R^{-1} \dot{q} + F_M + T_a = K_M V}
\end{cases}
$$

$$
\begin{cases}
\mathbf{M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + F_f(\dot{q}) + G(q) + J^T(q)F + T_P = T} \\
\mathbf{i_R^{-1}}\left(\mathbf{J_M} + \mathbf{J}_{ech}\right)\mathbf{\ddot{q}} + \mathbf{i_R^{-1}}\left(\mathbf{B} + \mathbf{b}_{ech}\right)\mathbf{\dot{q}} + \mathbf{F_M} + \mathbf{i_R^{-1} T} = \mathbf{K_M V}
\end{cases}
$$

$$\mathbf{i}_{\mathbf{R}}^{-1}\left(\mathbf{J}_{\mathbf{M}}+\mathbf{J}_{ech}+\mathbf{M}(\mathbf{q})\right)\ddot{\mathbf{q}}+\mathbf{i}_{\mathbf{R}}^{-1}\left(\mathbf{B}+\mathbf{b}_{ech}+\mathbf{V}_{\mathbf{m}}(\mathbf{q},\dot{\mathbf{q}})\right)\dot{\mathbf{q}}+\mathbf{F}_{\mathbf{M}}+\mathbf{i}_{\mathbf{R}}^{-1}\mathbf{F}_{\mathbf{f}}(\dot{\mathbf{q}})+\mathbf{i}_{\mathbf{R}}^{-1}\mathbf{G}(\mathbf{q})+\mathbf{i}_{\mathbf{R}}^{-1}\mathbf{J}^{\mathbf{T}}(\mathbf{q})\mathbf{F}+\mathbf{i}_{\mathbf{R}}^{-1}\mathbf{T}_{\mathbf{P}}=\mathbf{K}_{\mathbf{M}}\mathbf{V}$$

$$\mathbf{i}_{\mathbf{R}}^{-1}\downarrow\Rightarrow\begin{cases}\mathbf{i}_{\mathbf{R}}^{-1}\mathbf{M}\downarrow\\ \mathbf{i}_{\mathbf{R}}^{-1}\mathbf{V}_{\mathbf{m}}\downarrow\\ \mathbf{i}_{\mathbf{R}}^{-1}\mathbf{G}\downarrow\\ \mathbf{i}_{\mathbf{R}}^{-1}\mathbf{T}_{\mathbf{P}}\downarrow\end{cases}$$

CONCLUSIONS