The Manipulators models_3

The Kinematic Model

Agenda

- The Jacobian Definition
- Computing the Jacobian
- Using the Jacobian
- Joint variable

THE JACOBIAN DEFINITION

$$\theta \rightarrow x$$

$$\boldsymbol{\theta} + \delta\boldsymbol{\theta} \rightarrow \boldsymbol{x} + \delta\boldsymbol{x}$$

$$\delta\boldsymbol{\theta} \rightarrow \delta\boldsymbol{x}$$

$$\varepsilon_i = \begin{cases} 0 & \text{Revolute} \\ 1 & \text{Prismatic} \end{cases}$$

$$\overline{\varepsilon_i} = 1 - \varepsilon$$

$$q_i = \overline{\varepsilon_i}\theta_i + \varepsilon_i d_i$$

$$\boldsymbol{q} = [q_1, q_2, \cdots q_n]^T$$

$$\begin{Bmatrix} {}^{0}_{E}\boldsymbol{n} \\ {}^{0}_{E}\boldsymbol{o} \\ {}^{0}_{E}\boldsymbol{a} \\ {}^{0}\boldsymbol{p} \end{Bmatrix} = \boldsymbol{f}(q_1, q_2, \dots, q_n)$$

$$\delta x_1 = \frac{\partial f_1(\boldsymbol{q})}{\partial q_1} \cdot \delta q_1 + \cdots + \frac{\partial f_1(\boldsymbol{q})}{\partial q_n} \cdot \delta q_n$$

……………………………………

$$\delta x_m = \frac{\partial f_m(\boldsymbol{q})}{\partial q_1} \cdot \delta q_1 + \cdots + \frac{\partial f_m(\boldsymbol{q})}{\partial q_n} \cdot \delta q_n$$

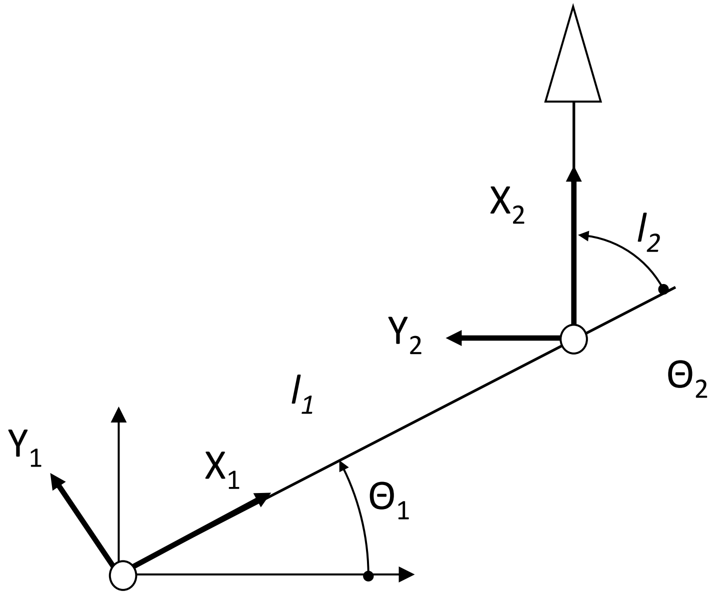
$$\delta \boldsymbol{x} = \begin{bmatrix} \delta x_1 \\ \vdots \\ \vdots \\ \delta x_m \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \cdots & \cdots & \frac{\partial f_1}{\partial q_n} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \frac{\partial f_m}{\partial q_1} & \cdots & \cdots & \frac{\partial f_m}{\partial q_n} \end{bmatrix} \begin{bmatrix} \delta q_1 \\ \vdots \\ \vdots \\ \delta q_n \end{bmatrix}$$

$$\delta \boldsymbol{x} = \begin{bmatrix} \delta x_1 \\ \vdots \\ \vdots \\ \delta x_m \end{bmatrix} = \begin{bmatrix} \frac{\partial f_1}{\partial q_1} & \cdots & \cdots & \frac{\partial f_1}{\partial q_n} \\ \vdots & & & \vdots \\ \vdots & & & \vdots \\ \frac{\partial f_m}{\partial q_1} & \cdots & \cdots & \frac{\partial f_m}{\partial q_n} \end{bmatrix} \begin{bmatrix} \delta q_1 \\ \vdots \\ \vdots \\ \delta q_n \end{bmatrix}$$

$$\delta \boldsymbol{x}_{(m\times 1)} = \boldsymbol{J}_{(m\times n)}(\boldsymbol{q}) \cdot \delta \boldsymbol{q}_{(n\times 1)}$$

$$\delta \boldsymbol{x}_{(m\times 1)} = \boldsymbol{J}_{(m\times n)}(\boldsymbol{q}) \cdot \delta \boldsymbol{q}_{(n\times 1)}$$

$$\boxed{\dot{\boldsymbol{x}}_{(m\times 1)} = \boldsymbol{J}_{(m\times n)}(\boldsymbol{q}) \cdot \dot{\boldsymbol{q}}_{(n\times 1)}}$$

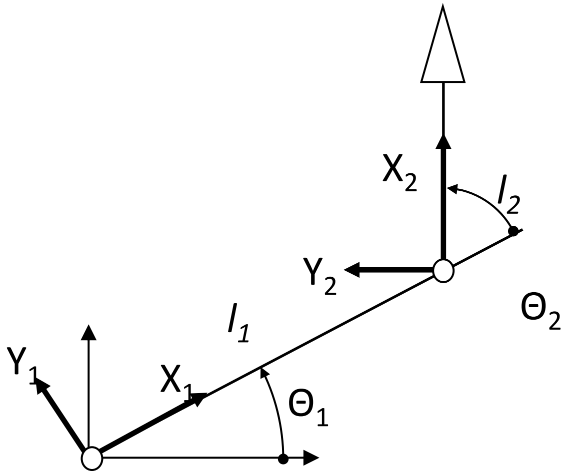


$$x = l_1 c_1 + l_2 c_{12}$$

$$y = l_1 s_1 + l_2 s_{12}$$

$$\delta x = -(l_1 s_1 + l_2 s_{12})\delta\theta_1 - l_2 s_{12}\delta\theta_2$$

$$\delta y = (l_1 c_1 + l_2 c_{12})\delta\theta_1 + l_2 c_{12}\delta\theta_2$$



$$\delta x = -(l_1 s_1 + l_2 s_{12})\delta\theta_1 - l_2 s_{12}\delta\theta_2$$

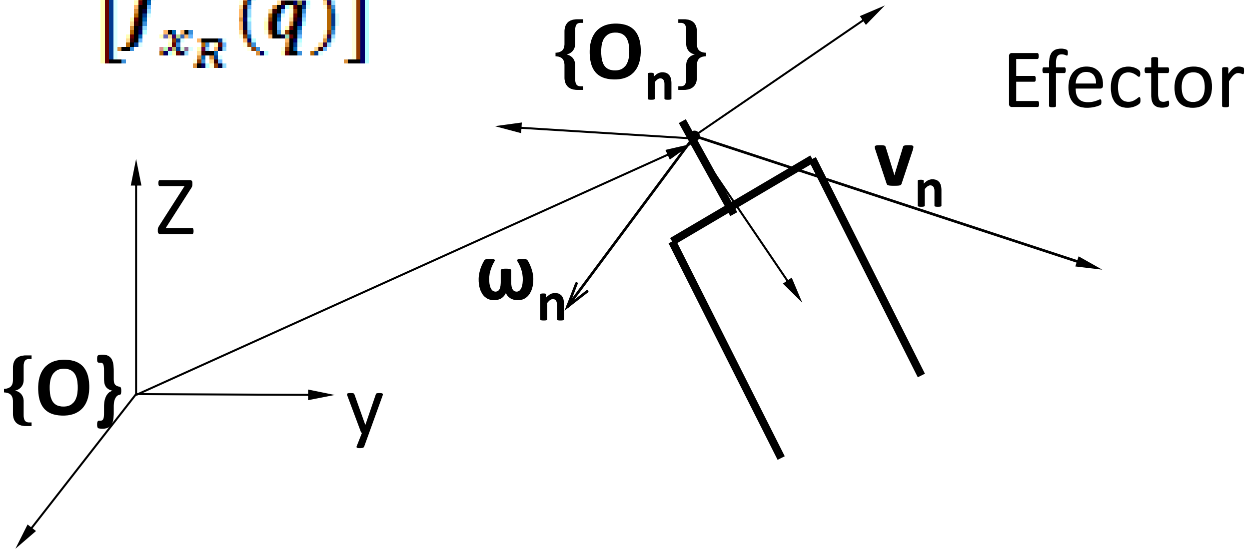
$$\delta y = (l_1 c_1 + l_2 c_{12})\delta\theta_1 + l_2 c_{12}\delta\theta_2$$

$$\delta x = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} \end{bmatrix} \cdot \begin{bmatrix} \delta\theta_1 \\ \delta\theta_2 \end{bmatrix}$$

$$J = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} \end{bmatrix}$$

$$
\begin{bmatrix} v_x \\ v_y \\ v_z \\ \omega_x \\ \omega_y \\ \omega_z \end{bmatrix} = \begin{bmatrix} J_{11} & & J_{1n} \\ & & \\ & & \\ & & \\ & & \\ J_{61} & & J_{6n} \end{bmatrix} \cdot \begin{bmatrix} \dot{q}_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \dot{q}_n \end{bmatrix}
$$

$$x = \begin{bmatrix} x_P \\ x_R \end{bmatrix} \qquad \begin{bmatrix} J_{x_P}(q) \\ J_{x_R}(q) \end{bmatrix}$$



$$\begin{bmatrix} v \\ \omega \end{bmatrix} = J_O(q)_{(6\times n)} \cdot \dot{q}_{(n\times 1)}$$

$$\dot{\boldsymbol{x}}_P = \boldsymbol{E}_P(\boldsymbol{x}_P) \cdot \boldsymbol{v}$$

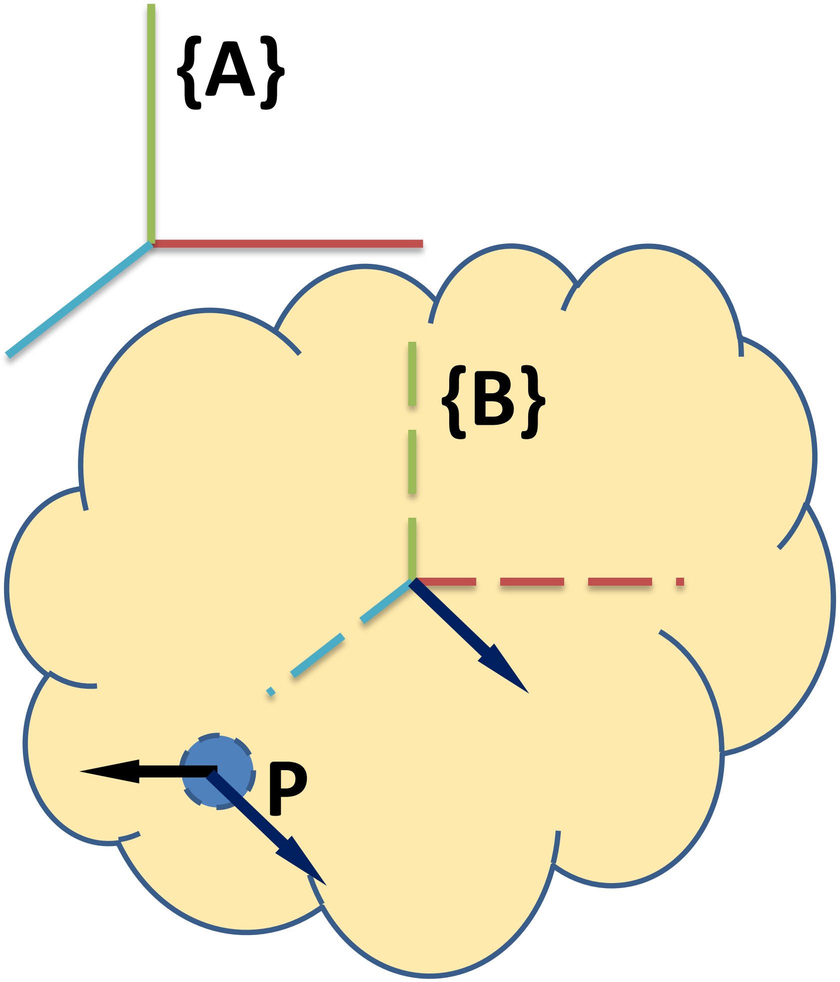
$$\dot{\boldsymbol{x}}_R = \boldsymbol{E}_R(\boldsymbol{x}_R) \cdot \boldsymbol{\omega}$$

$$\begin{cases} \boldsymbol{x}_P = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; \ \boldsymbol{E}_P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \\ \boldsymbol{x}_R = \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix}; \ \boldsymbol{E}_R = \begin{bmatrix} -\dfrac{s\alpha \cdot c\beta}{s\beta} & \dfrac{c\alpha \cdot c\beta}{s\beta} & 1 \\ c\alpha & s\alpha & 0 \\ \dfrac{s\alpha}{s\beta} & -\dfrac{c\alpha}{s\beta} & 0 \end{bmatrix} \end{cases}$$

$$J = \begin{bmatrix} J_{x_P} \\ J_{x_R} \end{bmatrix} = \begin{bmatrix} E_P & 0 \\ 0 & E_R \end{bmatrix} \cdot \begin{bmatrix} J_v \\ J_\omega \end{bmatrix}$$

$$J(q) = E(x) \cdot J_O$$

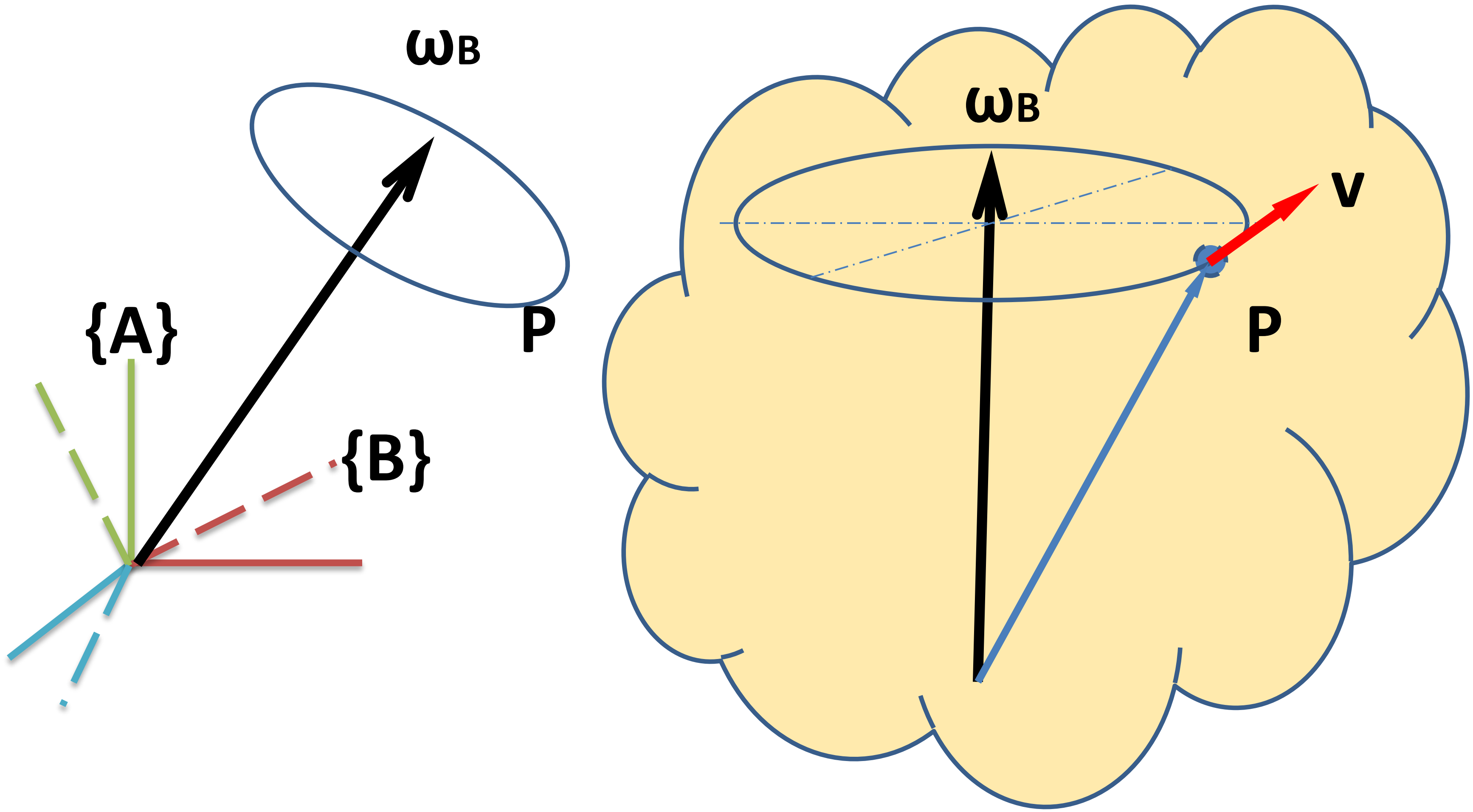
COMPUTING THE JACOBIAN - 1



$$^{A}\bar{v}_{P} = {}^{A}\bar{v}_{B} + {}^{B}\bar{v}_{P}$$

$$^{A}\bar{v}_{P} = {}^{A}v_{B} + {}^{A}_{B}R\, {}^{B}v_{P}$$

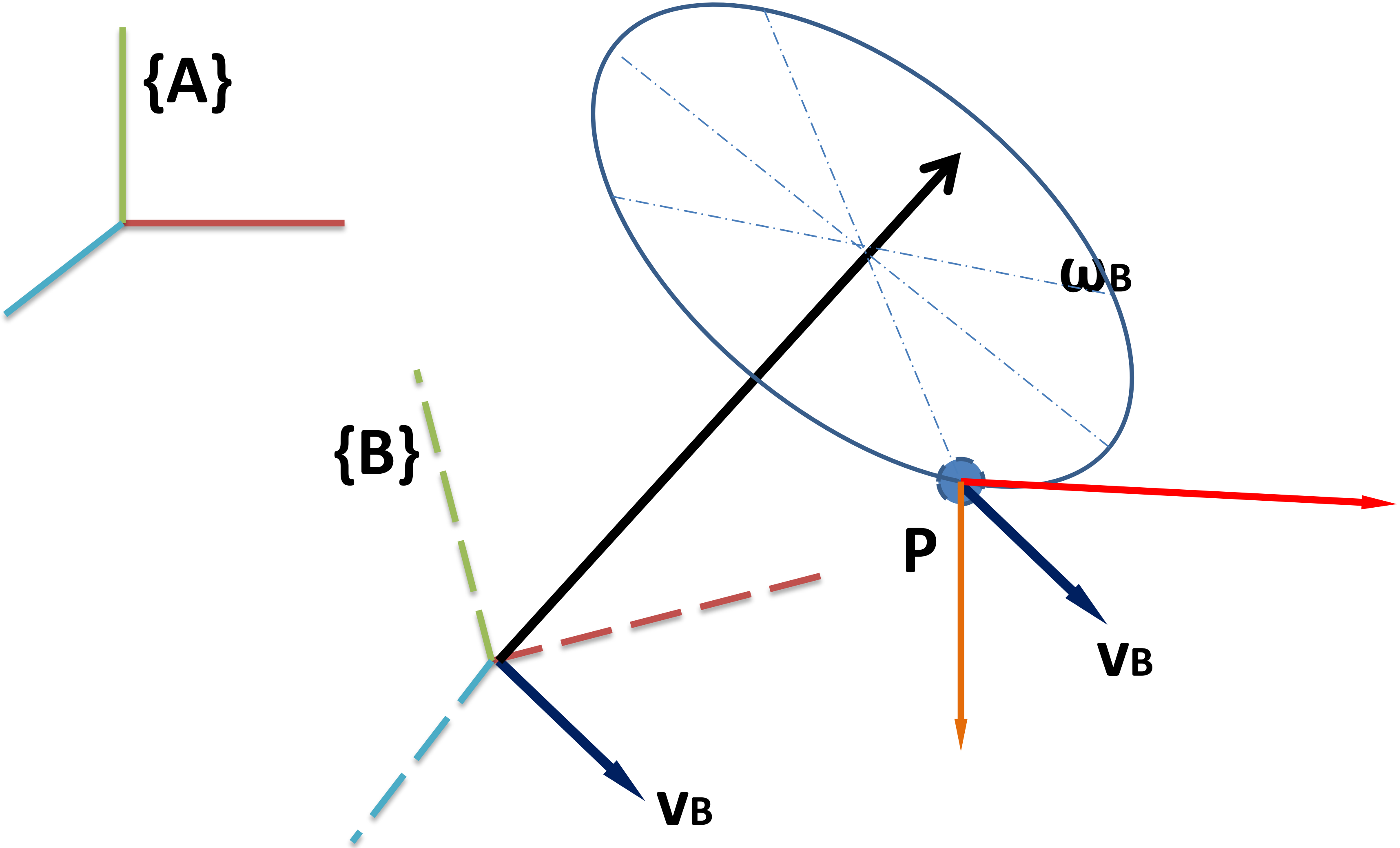
$$^{A}_{B}R = I$$



$$^{A}v_{P} = \Omega \times P$$

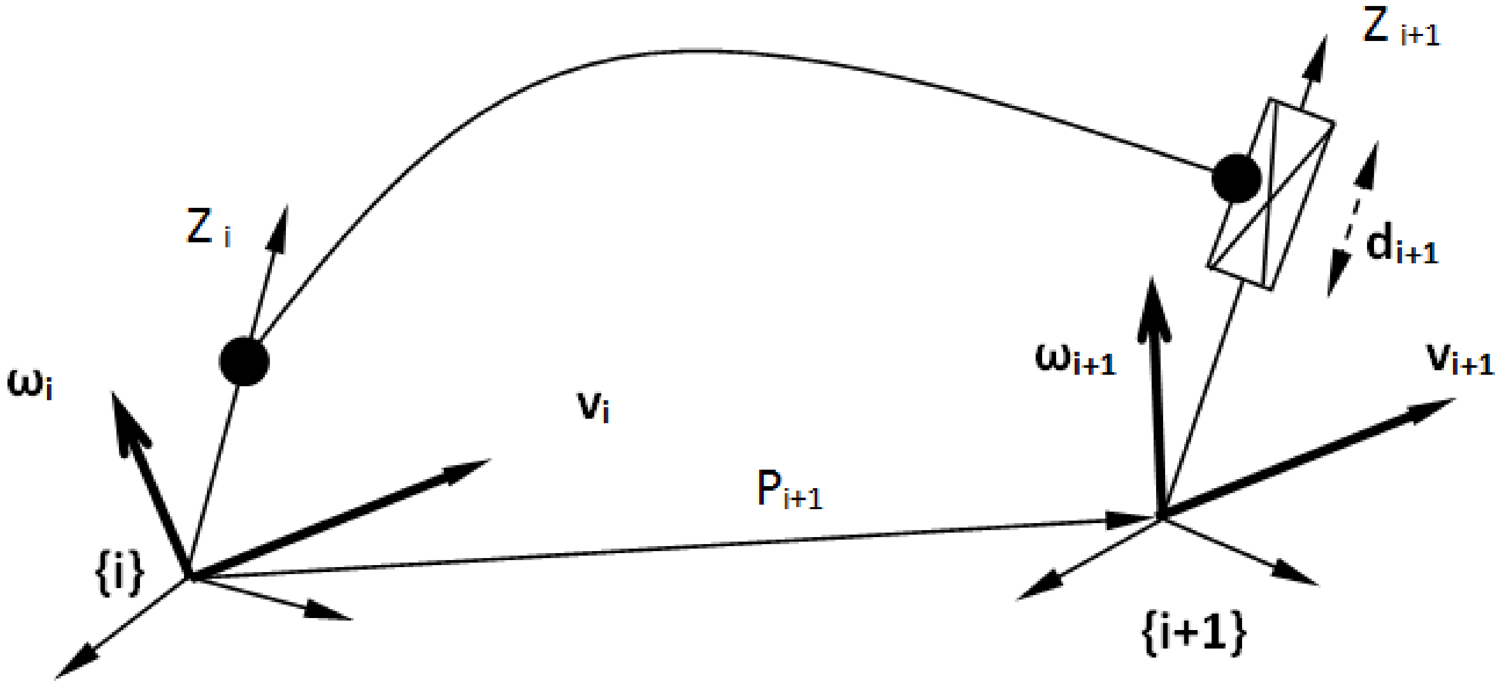
$$ {}^{A}\boldsymbol{v}_{P} = \boldsymbol{\Omega} \times \boldsymbol{P} \quad \Longleftrightarrow \quad {}^{A}\boldsymbol{v}_{P} = \tilde{\boldsymbol{\Omega}} P $$

$$ \boldsymbol{\Omega} = \begin{bmatrix} \Omega_x \\ \Omega_y \\ \Omega_z \end{bmatrix} \qquad \tilde{\boldsymbol{\Omega}} = \begin{bmatrix} 0 & -\Omega_z & \Omega_y \\ \Omega_z & 0 & -\Omega_x \\ -\Omega_y & \Omega_x & 0 \end{bmatrix} $$



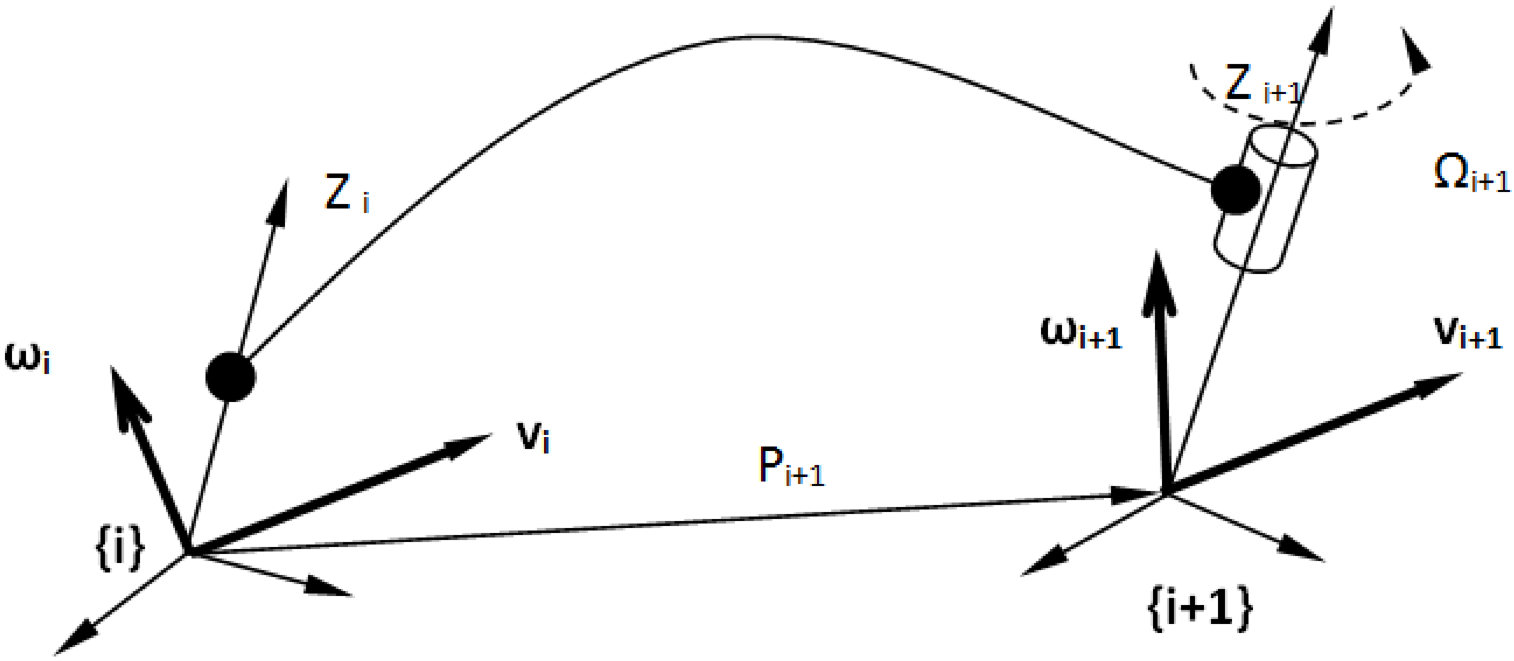
$$ {}^{A}v_{P} = {}^{A}v_{B} + {}^{A}_{B}R \cdot {}^{B}v_{P} + {}^{A}\tilde{\Omega}_{B} \cdot {}^{A}_{B}R\, {}^{B}P_{B} $$

$$\frac{d\left(\alpha\hat{\mathbf{x}}\right)}{dt} = \dot{\alpha}\hat{\mathbf{x}} + \boldsymbol{\omega} \times \alpha\hat{\mathbf{x}}$$



$$v_{i+1} = v_i + \omega_i \times P_{i+1} + \dot{d}_{i+1} \cdot Z_{i+1}$$

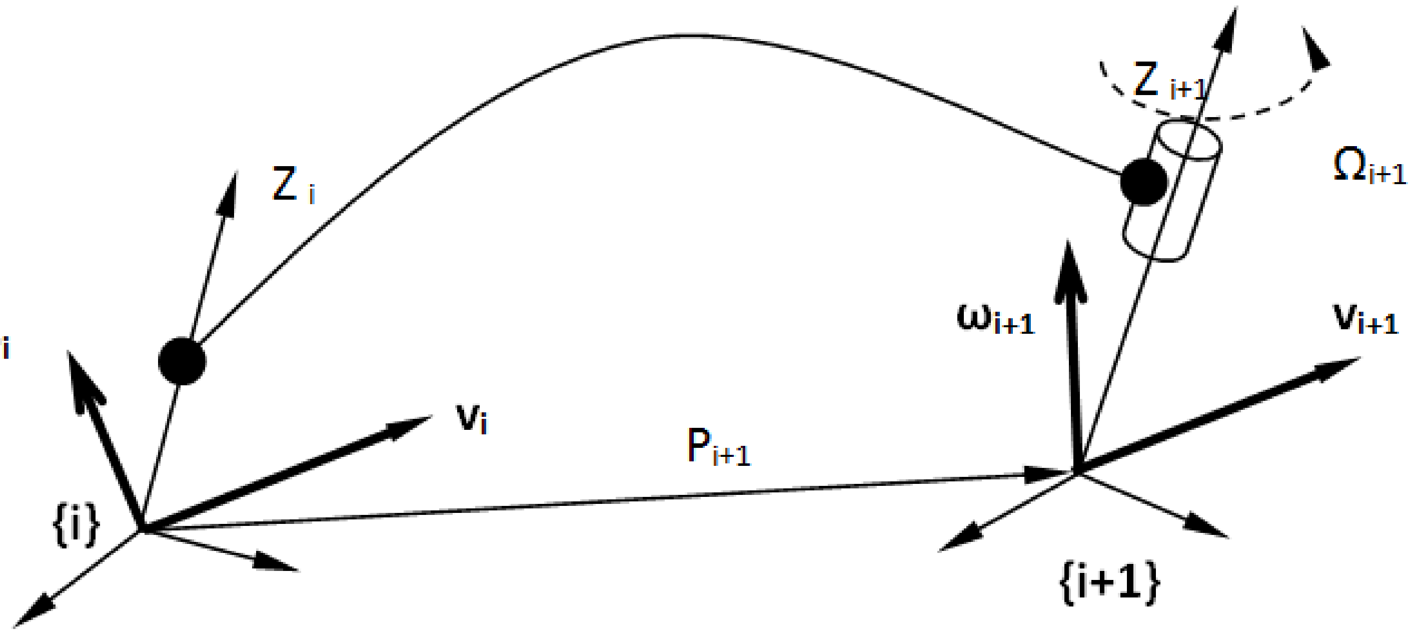
$$\omega_{i+1} = \omega_i$$



$$v_{i+1} = v_i + \omega_i \times P_{i+1}$$

$$\omega_{i+1} = \omega_i + \Omega_{i+1}$$

$$\Omega_{i+1} = \dot{\theta}_{i+1} \cdot Z_{i+1}$$



$$\boldsymbol{v}_{i+1} = \boldsymbol{v}_i + \boldsymbol{\omega}_i \times \boldsymbol{P}_{i+1} + \dot{d}_{i+1} \cdot \boldsymbol{Z}_{i+1}$$

$$\boldsymbol{\omega}_{i+1} = \boldsymbol{\omega}_i + \dot{\theta}_{i+1} \cdot \boldsymbol{Z}_{i+1}$$

Joint {1}: $^{1}\boldsymbol{v}$, $^{1}\boldsymbol{\omega}$

Joint $\{2\}$: **We know**:

- $^{1}\boldsymbol{v}$, $^{1}\boldsymbol{\omega}$;
- $^{2}_{1}\boldsymbol{R}$ $^{1}\boldsymbol{p}_2$ $^{2}\boldsymbol{Z}$
- $\dot{\theta}_2$ $\dot{d}_2$

We compute:

- $^{2}\boldsymbol{v}$, $^{2}\boldsymbol{\omega}$

$$^{2}\boldsymbol{\omega} = {}^{2}_{1}\boldsymbol{R} \cdot {}^{1}\boldsymbol{\omega} + \dot{\theta}_2 \cdot {}^{2}\boldsymbol{Z}$$

$$^{2}\boldsymbol{v} = {}^{2}_{1}\boldsymbol{R}({}^{1}\boldsymbol{v} + {}^{1}\boldsymbol{\omega} \times {}^{1}\boldsymbol{p}_2) + \dot{d}_2 \cdot {}^{2}\boldsymbol{Z}$$

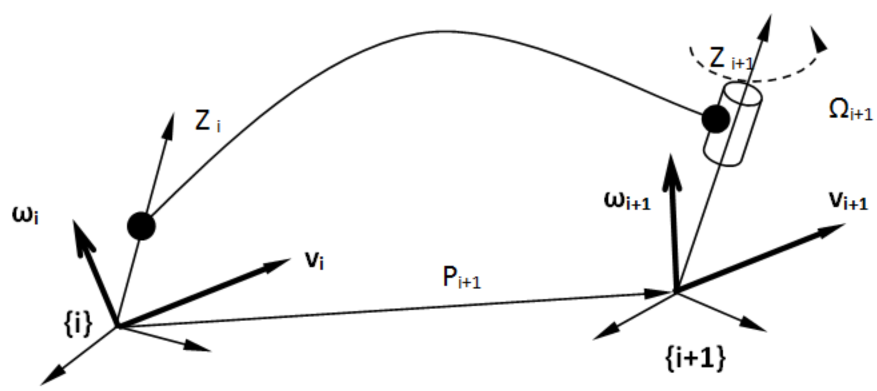
Joint $\{i+1\}$:

We know:

- ${}^{i}\boldsymbol{v}$, ${}^{i}\boldsymbol{\omega}$;
- ${}^{i+1}_{i}\boldsymbol{R}$ ${}^{i}\boldsymbol{p}_{i+1}$ ${}^{i+1}\boldsymbol{Z}$;
- $\dot{\theta}_{i+1}$ $\dot{d}_{i+1}$

We compute:

- ${}^{i+1}\boldsymbol{v}$, ${}^{i+1}\boldsymbol{\omega}$



$$ {}^{i+1}\boldsymbol{\omega} = {}^{i+1}_{i}\boldsymbol{R} \cdot {}^{i}\boldsymbol{\omega} + \dot{\theta}_{i+1} \cdot {}^{i+1}\boldsymbol{Z} $$

$$ {}^{i+1}\boldsymbol{v} = {}^{i+1}_{i}\boldsymbol{R}\left({}^{i}\boldsymbol{v} + {}^{i}\boldsymbol{\omega} \times {}^{i}\boldsymbol{p}_{i+1}\right) + \dot{d}_{i+1} \cdot {}^{i+1}\boldsymbol{Z} $$

Joint $\{\boldsymbol{n}\}$: **We know**:

- $^{n-1}\boldsymbol{v}$, $^{n-1}\boldsymbol{\omega}$
- ${}^{n}_{n-1}\boldsymbol{R}$ $^{n-1}\boldsymbol{p}_n$ $^{n}\boldsymbol{Z}$
- $\dot{\theta}_n$ $\dot{d}_n$

We compute:

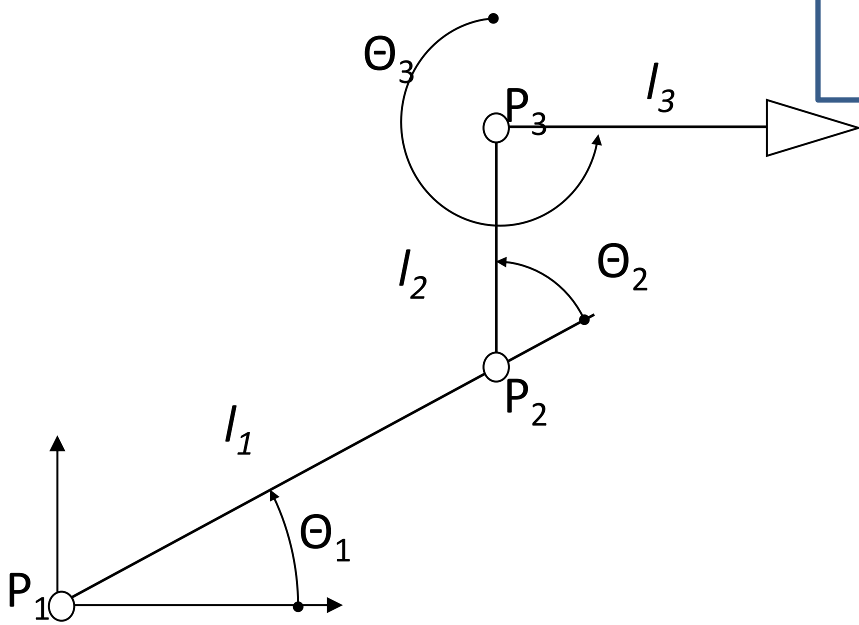
- $^{n}\boldsymbol{v}$, $^{n}\boldsymbol{\omega}$

$$^{n}\boldsymbol{\omega} = {}^{n}_{n-1}\boldsymbol{R} \cdot {}^{n-1}\boldsymbol{\omega} + \dot{\theta}_n \cdot {}^{n}\boldsymbol{Z}$$

$$^{n}\boldsymbol{v} = {}^{n}_{n-1}\boldsymbol{R}({}^{n-1}\boldsymbol{v} + {}^{n-1}\boldsymbol{\omega} \times {}^{n-1}\boldsymbol{p}_n) + \dot{d}_n \cdot {}^{n}\boldsymbol{Z}$$

$$\begin{bmatrix} {}^{o}\boldsymbol{v}_n \\ {}^{o}\boldsymbol{\omega}_n \end{bmatrix} = \begin{bmatrix} {}^{o}_{n}\boldsymbol{R} & 0 \\ 0 & {}^{o}_{n}\boldsymbol{R} \end{bmatrix} \cdot \begin{bmatrix} {}^{n}\boldsymbol{v}_n \\ {}^{n}\boldsymbol{\omega}_n \end{bmatrix}$$

;

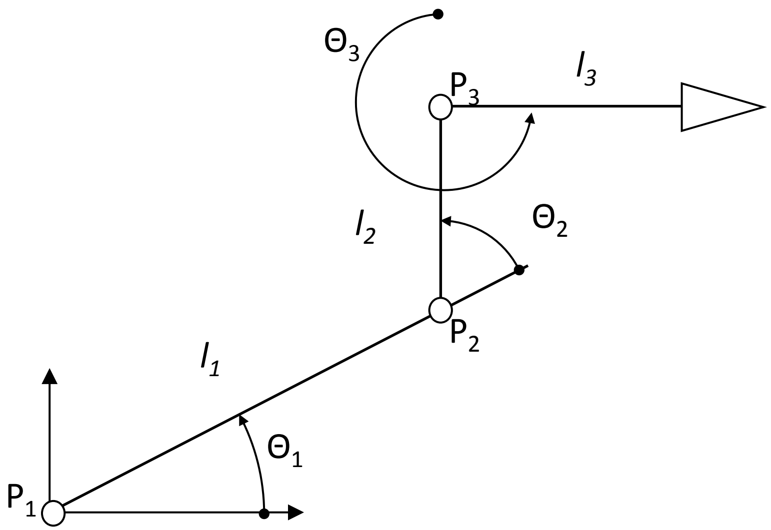


$$v_{i+1} = v_i + \omega_i \times P_{i+1} + \dot{d}_{i+1} \cdot Z_{i+1}$$

$$\begin{cases} \boldsymbol{v}_{P_1} = 0 \\ \boldsymbol{v}_{P_2} = \boldsymbol{v}_{P_1} + \boldsymbol{\omega}_1 \times \boldsymbol{p}_2 \\ \boldsymbol{v}_{P_3} = \boldsymbol{v}_{P2} + \boldsymbol{\omega}_2 \times \boldsymbol{p}_3 \end{cases}$$

$$\begin{cases} \boldsymbol{\omega}_1 \equiv \boldsymbol{\Omega}_1 = \dot{\theta}_1 \cdot {}^{O}\boldsymbol{Z}_1 \\ \boldsymbol{\omega}_2 = \boldsymbol{\omega}_1 + \dot{\theta}_2 \cdot {}^{O}\boldsymbol{Z}_2 \\ \boldsymbol{\omega}_3 = \boldsymbol{\omega}_2 + \dot{\theta}_3 \cdot {}^{O}\boldsymbol{Z}_3 \end{cases}$$

;



$$\boldsymbol{v}_{P_2} = \boldsymbol{v}_{P_1} + \boldsymbol{\omega}_1 \times \boldsymbol{p}_2$$

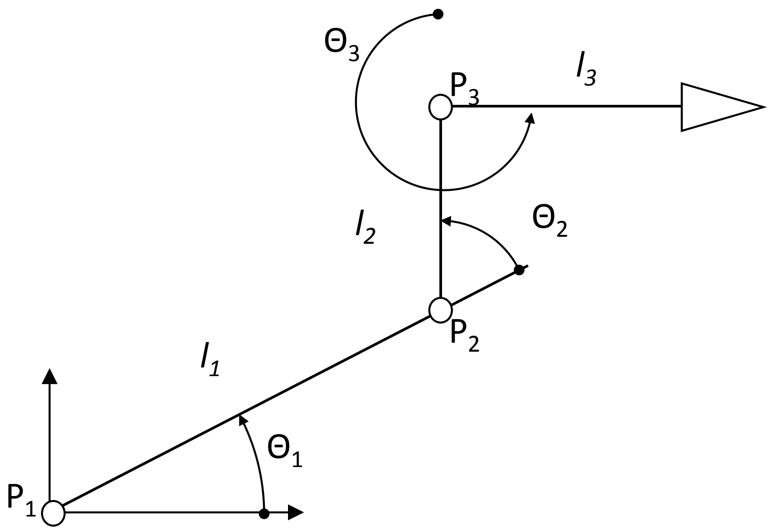
$$\boldsymbol{\omega}_2 = \boldsymbol{\omega}_1 + \dot{\theta}_2 \cdot {}^{O}\boldsymbol{Z}_2$$

$$\begin{bmatrix} l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} = {}^{O}_{1}\boldsymbol{R} \cdot [l_1 \quad 0 \quad 0]^T$$

$${}^{O}\boldsymbol{v}_{P_2} = 0 + \begin{bmatrix} 0 & -\dot{\theta}_1 & 0 \\ \dot{\theta}_1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} l_1 c_1 \\ l_1 s_1 \\ 0 \end{bmatrix} = \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1$$

$${}^{O}\boldsymbol{\omega}_2 = \left(\dot{\theta}_1 + \dot{\theta}_2\right) \cdot {}^{O}\boldsymbol{Z}_O$$

;



$$\boldsymbol{v}_{P_3} = \boldsymbol{v}_2 + \boldsymbol{\omega}_2 \times \boldsymbol{p}_3$$

$$\boldsymbol{\omega}_3 = \boldsymbol{\omega}_2 + \dot{\theta}_3 \cdot {}^{O}\boldsymbol{Z}_3$$

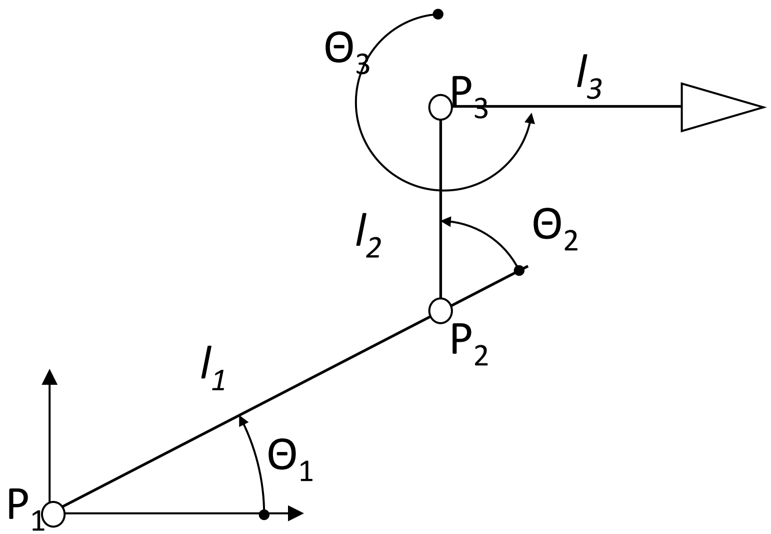
$${}^{O}\boldsymbol{v}_{P_2} = \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1$$

$$\begin{bmatrix} l_2 c_{12} \\ l_2 s_{12} \\ 0 \end{bmatrix} = {}^{O}_{2}\boldsymbol{R} \cdot [l_2 \quad 0 \quad 0]^T$$

$${}^{O}\boldsymbol{v}_{P_3} = \begin{bmatrix} -l_1 s_1 \\ l_1 c_1 \\ 0 \end{bmatrix} \dot{\theta}_1 + \begin{bmatrix} 0 & -(\dot{\theta}_1 + \dot{\theta}_2) & 0 \\ (\dot{\theta}_1 + \dot{\theta}_2) & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} l_2 c_{12} \\ l_2 s_{12} \\ 0 \end{bmatrix}$$

$${}^{O}\boldsymbol{v}_{P_3} = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

;



$$ {}^{O}\boldsymbol{v}_{P_3} = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} $$

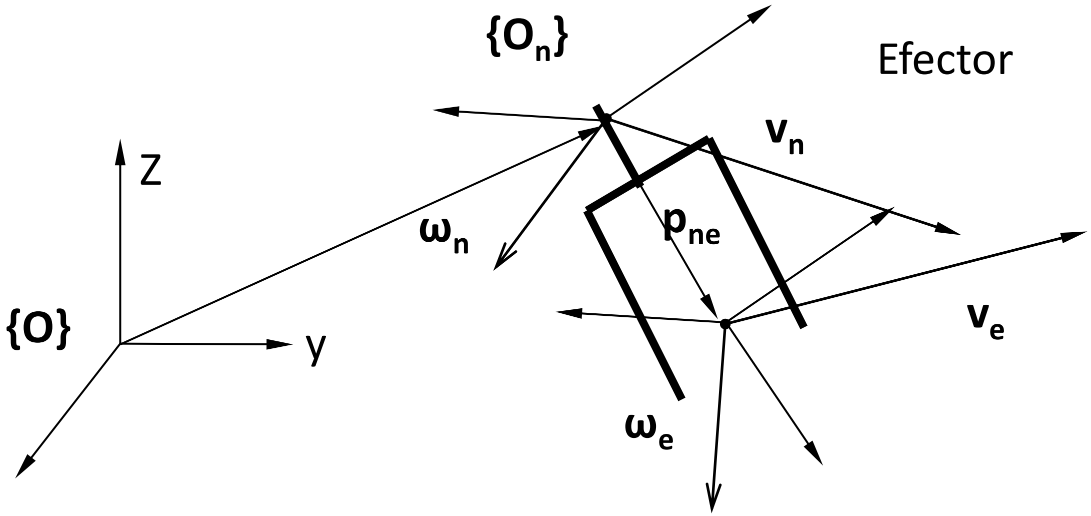
$$ {}^{O}\boldsymbol{\omega}_3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} $$

$$\begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} & 0 \\ 0 & 0 & 0 \end{bmatrix} = J_v$$

$$\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} = J_\omega$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} {}^O\boldsymbol{v}_{P_3} \\ {}^O\boldsymbol{\omega}_3 \end{bmatrix} = \begin{bmatrix} {}^O v_{P_3,x} \\ {}^O v_{P_3,y} \\ {}^O v_{P_3,z} \\ {}^O \omega_{3,x} \\ {}^O \omega_{3,y} \\ {}^O \omega_{3,z} \end{bmatrix} = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix}$$

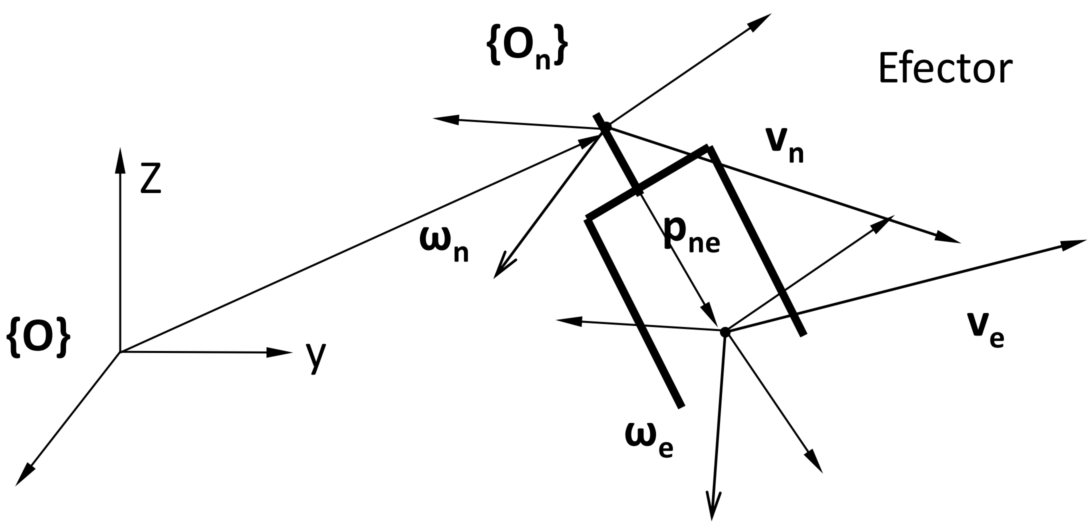
$$= \boldsymbol{J}_O \cdot \dot{\boldsymbol{q}}$$



$$\begin{cases} \boldsymbol{v}_e = \boldsymbol{v}_n + \boldsymbol{\omega}_n \times \boldsymbol{p}_{ne} \\ \boldsymbol{\omega}_e = \boldsymbol{\omega}_n \end{cases} \qquad \begin{cases} \boldsymbol{v}_e = \boldsymbol{v}_n - \boldsymbol{p}_{ne} \times \boldsymbol{\omega}_n \\ \boldsymbol{\omega}_e = \boldsymbol{\omega}_n \end{cases}$$

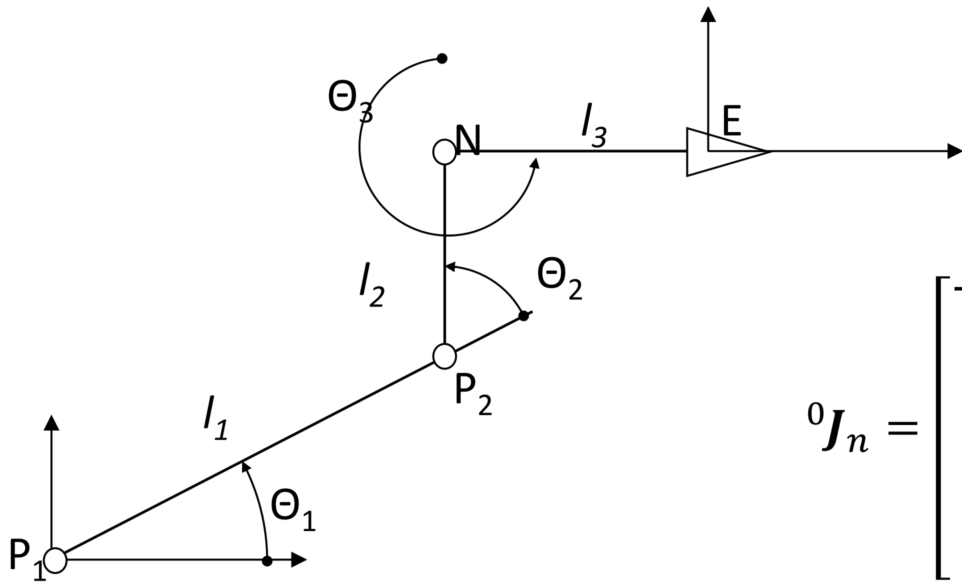
$$\begin{bmatrix} \boldsymbol{v}_e \\ \boldsymbol{\omega}_e \end{bmatrix} = \begin{bmatrix} \boldsymbol{I} & -\widetilde{\boldsymbol{p}}_{ne} \\ 0 & \boldsymbol{I} \end{bmatrix} \begin{bmatrix} \boldsymbol{v}_n \\ \boldsymbol{\omega}_n \end{bmatrix}$$

$$\boldsymbol{J}_e = \begin{bmatrix} \boldsymbol{I} & -\widetilde{\boldsymbol{p}}_{ne} \\ 0 & \boldsymbol{I} \end{bmatrix} \boldsymbol{J}_n$$



$$J_e = \begin{bmatrix} I & -\widetilde{p}_{ne} \\ 0 & I \end{bmatrix} J_n \qquad {}^{O}\widetilde{p} = {}^{O}_{n}R \cdot {}^{n}\widetilde{p} \cdot {}^{O}_{n}R^{T} \qquad {}^{i}J = \begin{bmatrix} {}^{i}_{j}R & 0 \\ 0 & {}^{i}_{j}R \end{bmatrix} {}^{j}J$$

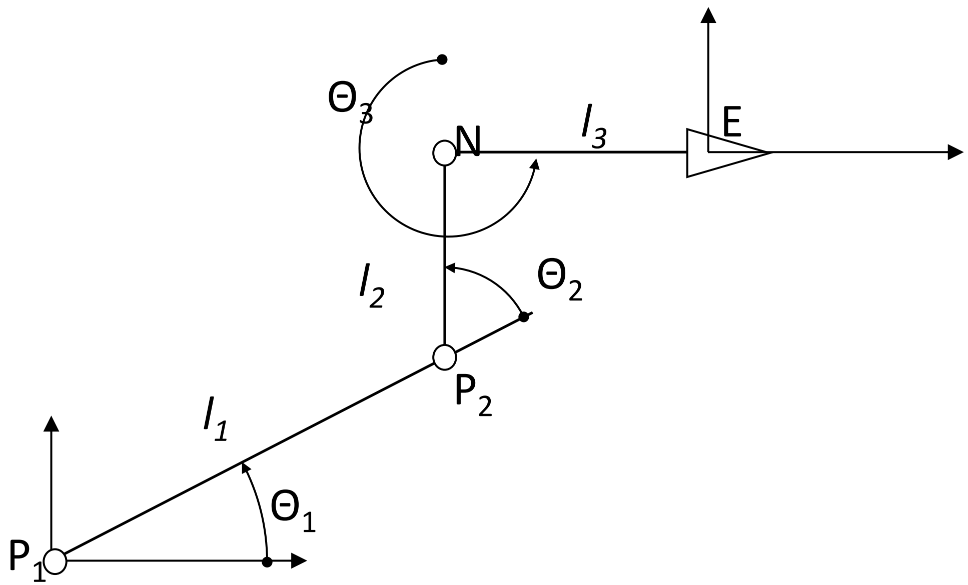
$${}^{O}J_e = \begin{bmatrix} {}^{O}_{n}R & -{}^{O}_{n}R \cdot {}^{n}\widetilde{p} \cdot {}^{O}_{n}R^{T} \\ 0 & {}^{O}_{n}R \end{bmatrix} {}^{n}J_n$$



$$ {}^{0}\boldsymbol{J}_n = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} $$

$$ {}^{O}\boldsymbol{p}_{ne} = \begin{bmatrix} l_3 c_{123} \\ l_3 s_{123} \\ 0 \end{bmatrix} \qquad \tilde{\boldsymbol{p}}_{ne} = \begin{bmatrix} 0 & 0 & l_3 s_{123} \\ 0 & 0 & -l_3 c_{123} \\ -l_3 s_{123} & l_3 c_{123} & 0 \end{bmatrix} $$

$$ {}^{O}\boldsymbol{J}_e = \begin{bmatrix} 1 & -{}^{O}\tilde{\boldsymbol{p}}_{ne} \\ 0 & 1 \end{bmatrix} {}^{O}\boldsymbol{J}_n $$

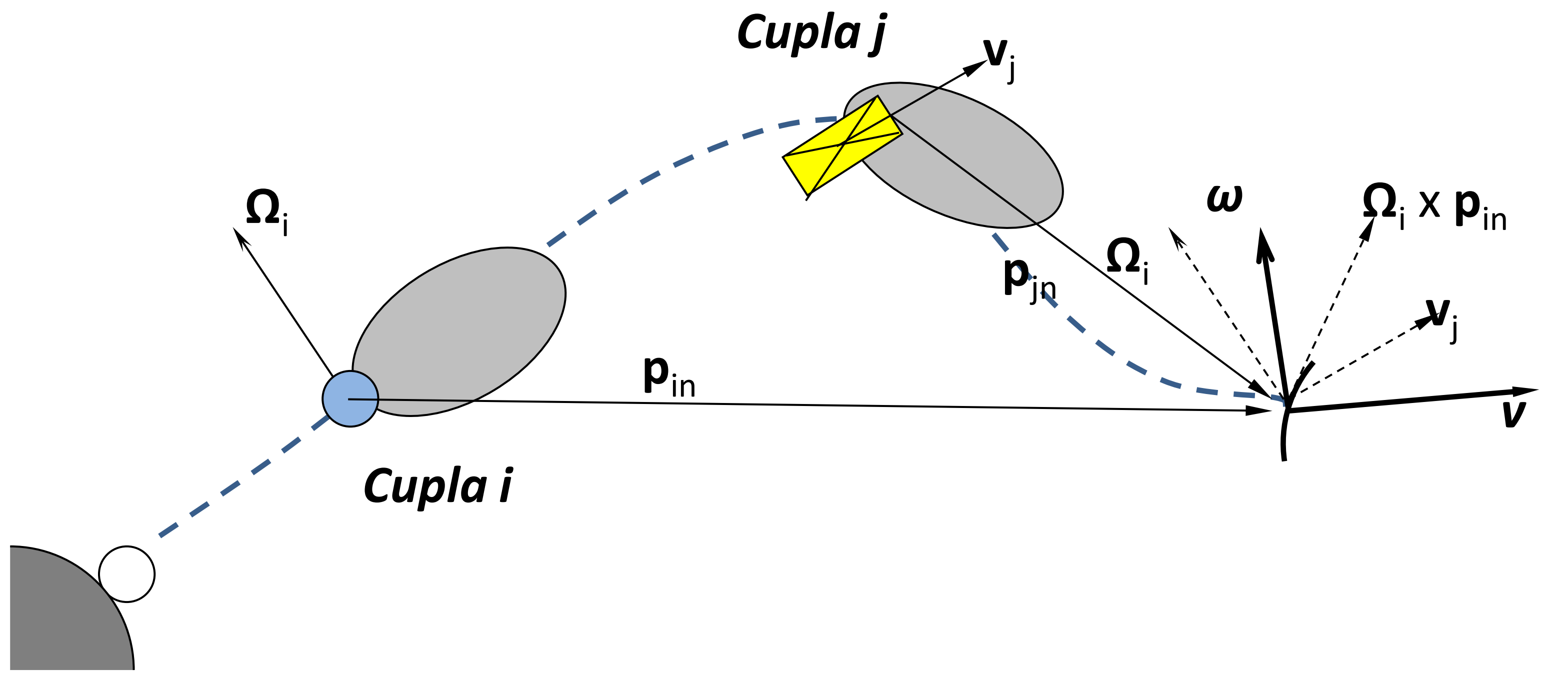


$$
{}^{0}\boldsymbol{J}_n = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} & -l_2 s_{12} & 0 \\ l_1 c_1 + l_2 c_{12} & l_2 c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}
$$

$$
{}^{O}\boldsymbol{J}_e = \begin{bmatrix} 1 & -{}^{O}\tilde{\boldsymbol{p}}_{ne} \\ 0 & 1 \end{bmatrix} {}^{O}\boldsymbol{J}_n
$$

$$
{}^{O}J_E = \begin{bmatrix} -l_1 s_1 - l_2 s_{12} - l_3 s_{123} & -l_2 s_{12} - l_3 s_{123} & -l_3 s_{123} \\ l_1 c_1 + l_2 c_{12} + l_3 c_{123} & l_2 c_{12} + l_3 c_{123} & l_3 c_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}
$$

COMPUTING THE JACOBIAN - 2



	Prismatic Joint	Revolute Joint
Linear speed	v_j	$\Omega_i \times P_{in}$
Angular Speed		Ω_i

;

$$\begin{cases} \boldsymbol{v} = \sum_{i=1}^{n} [\varepsilon_i \boldsymbol{v}_i + \bar{\varepsilon}_i (\boldsymbol{\Omega}_i \times \boldsymbol{P}_{in})] \\ \boldsymbol{\omega} = \sum_{i=1}^{n} \bar{\varepsilon}_i \boldsymbol{\Omega}_i \end{cases}$$

$$\boldsymbol{v}_i = \mathbf{Z}_i \cdot \dot{q}_i \qquad \boldsymbol{\Omega}_i = \mathbf{Z}_i \cdot \dot{q}_i$$

$$\begin{cases} \boldsymbol{v} = \sum_{i=1}^{n} [\epsilon_i \boldsymbol{Z}_i + \bar{\epsilon}_i (\boldsymbol{Z}_i \times \boldsymbol{P}_{in})] \cdot \dot{q}_i \\ \boldsymbol{\omega} = \sum_{i=1}^{n} [\bar{\epsilon}_i \boldsymbol{Z}_i] \cdot \dot{q}_i \end{cases}$$

$$\boldsymbol{v} = [\epsilon_1 \boldsymbol{Z}_1 + \bar{\epsilon}_1 (\boldsymbol{Z}_1 \times \boldsymbol{P}_{1n}) \quad \epsilon_2 \boldsymbol{Z}_2 + \bar{\epsilon}_2 (\boldsymbol{Z}_2 \times \boldsymbol{P}_{2n}) \quad \ldots] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$$\boldsymbol{v} = \boldsymbol{J}_v \cdot \dot{\boldsymbol{q}}$$

$$\boldsymbol{\omega} = [\bar{\epsilon}_1 \boldsymbol{Z}_1 \quad \bar{\epsilon}_2 \boldsymbol{Z}_2 \quad \dots] \begin{bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \vdots \\ \dot{q}_n \end{bmatrix}$$

$$\boldsymbol{\omega} = \boldsymbol{J}_{\omega} \cdot \dot{\boldsymbol{q}}$$

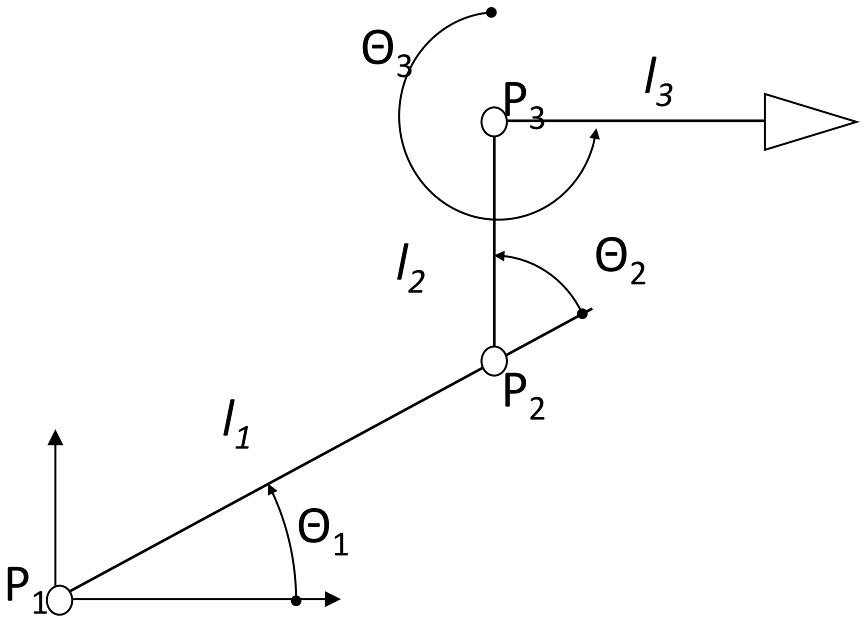
$$\boldsymbol{J} = \begin{bmatrix} \boldsymbol{J}_v \\ \hline \boldsymbol{J}_{\omega} \end{bmatrix}$$

$$\boldsymbol{v} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix} = \dot{\boldsymbol{x}}_P = \frac{\partial \boldsymbol{x}_P}{\partial q_1} \cdot \dot{q}_1 + \cdots + \frac{\partial \boldsymbol{x}_P}{\partial q_n} \cdot \dot{q}_n$$

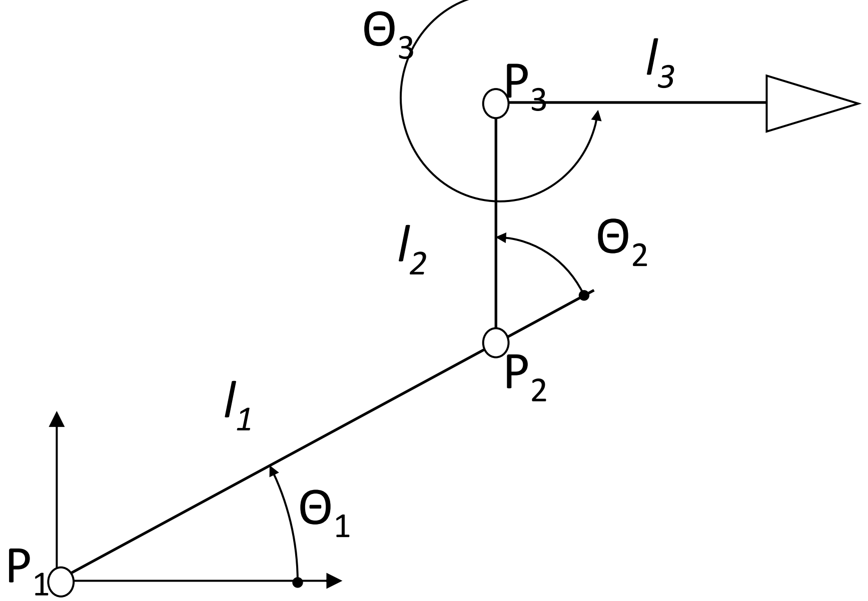
$$\boldsymbol{J}_v = \begin{bmatrix} \frac{\partial \boldsymbol{x}_P}{\partial q_1} & \frac{\partial \boldsymbol{x}_P}{\partial q_2} & \cdots & \frac{\partial \boldsymbol{x}_P}{\partial q_n} \end{bmatrix}$$

$$
{}^{o}\boldsymbol{J} = \begin{bmatrix} \dfrac{\partial\, {}^{o}\boldsymbol{x}_P}{\partial q_1} & \dfrac{\partial\, {}^{o}\boldsymbol{x}_P}{\partial q_2} & \cdots & \dfrac{\partial\, {}^{o}\boldsymbol{x}_P}{\partial q_n} \\ \bar{\epsilon}_1\, {}^{o}\mathbf{Z}_1 & \bar{\epsilon}_2\, {}^{o}\mathbf{Z}_2 & \cdots & \bar{\epsilon}_n\, {}^{o}\mathbf{Z}_n \end{bmatrix}
$$

$$
{}^{o}\mathbf{Z}_i = {}^{o}_{i}\boldsymbol{R}\; {}^{i}\mathbf{Z}_i
$$



$$\dot{\boldsymbol{x}} = \begin{bmatrix} {}^{O}\boldsymbol{v}_{P_3} \\ {}^{O}\boldsymbol{\omega}_3 \end{bmatrix} = \begin{bmatrix} {}^{O}v_{P_3,x} \\ {}^{O}v_{P_3,y} \\ {}^{O}v_{P_3,z} \\ {}^{O}\omega_{3,x} \\ {}^{O}\omega_{3,y} \\ {}^{O}\omega_{3,z} \end{bmatrix} = \begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix} \cdot \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \\ \dot{\theta}_3 \end{bmatrix} = \boldsymbol{J}_O \cdot \dot{\boldsymbol{q}}$$



$$\begin{bmatrix} -(l_1 s_1 + l_2 s_{12}) & -l_2 s_{12} & 0 \\ (l_1 c_1 + l_2 c_{12}) & l_2 c_{12} & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

COMPUTING THE JACOBIAN - 3

$$ {}^{0}\mathbf{J} = \begin{bmatrix} \dfrac{\partial {}^{0}\mathbf{x}_P}{\partial q_1}, & \ldots, & \dfrac{\partial {}^{0}\mathbf{x}_P}{\partial q_n} \\ \bar{\varepsilon}_1 {}^{0}\mathbf{Z}_1, & \ldots, & \bar{\varepsilon}_n {}^{0}\mathbf{Z}_n \end{bmatrix} $$

$$ {}^{0}\mathbf{J} \approx \begin{bmatrix} \dfrac{\Delta {}^{0}\mathbf{x}_P(\mathbf{q} + \Delta q_1)}{\Delta q_1}, & \ldots, & \dfrac{\Delta {}^{0}\mathbf{x}_P(\mathbf{q} + \Delta q_n)}{\Delta q_n} \\ \bar{\varepsilon}_1 {}^{0}\mathbf{Z}_1, & \ldots, & \bar{\varepsilon}_n {}^{0}\mathbf{Z}_n \end{bmatrix} $$

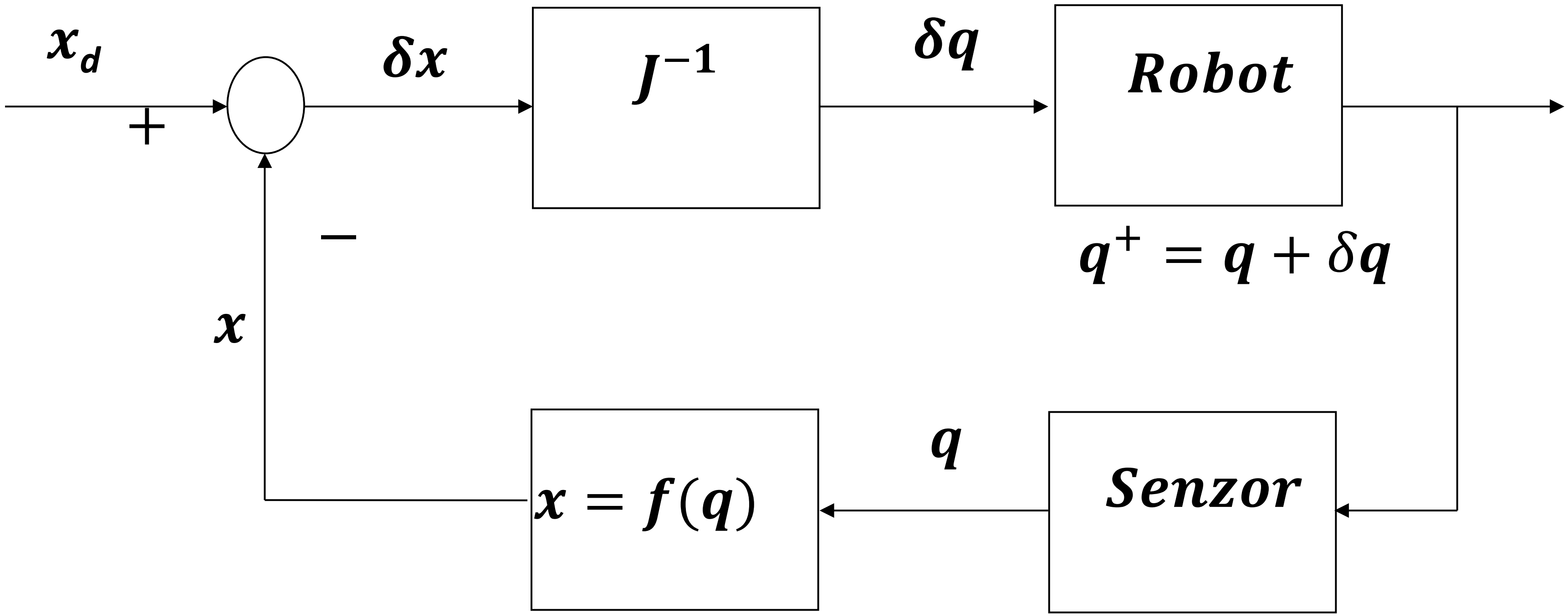
$$ \Delta {}^{0}\mathbf{x}(\mathbf{q} + \Delta q_i) = {}^{0}\mathbf{x}(\mathbf{q} + \Delta q_i) - {}^{0}\mathbf{x}(\mathbf{q}) $$

$$ (\mathbf{q} + \Delta q_i) = \begin{bmatrix} q_1 & .. & q_i + \Delta q_i & \ldots & q_n \end{bmatrix}^T $$

USING THE JACOBIAN FOR CONTROL

$$\delta \boldsymbol{x} = \boldsymbol{x}_d - \boldsymbol{x}$$

$$\delta \boldsymbol{q} = \boldsymbol{J}^{-1} \delta \boldsymbol{x}$$

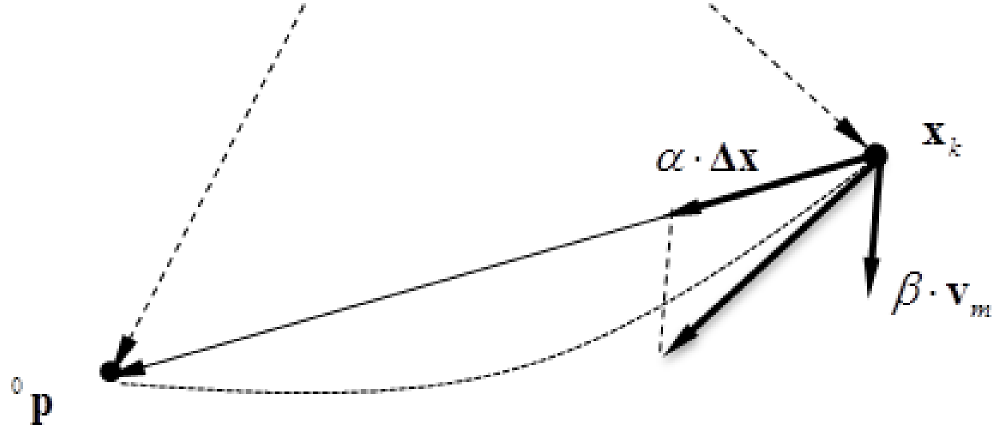


USING THE JACOBIAN FOR THE SOLVING THE INVERSE GEOMETRY

$$\begin{cases} \mathbf{q}_P = \mathbf{f}_P^{-1}\left({}^0\mathbf{p}\right) \\ \mathbf{q}_O = \mathbf{f}_O^{-1}\left(\begin{bmatrix} {}^0_E\mathbf{n} \\ {}^0_E\mathbf{o} \\ {}^0_E\mathbf{a} \end{bmatrix}\right) \end{cases}$$

$$\mathbf{\Delta x} \approx \mathbf{J(q)} \cdot \mathbf{\Delta q}$$

$$\mathbf{\Delta q} \approx \mathbf{J}^{-1}\mathbf{(q)} \cdot \mathbf{\Delta x}$$

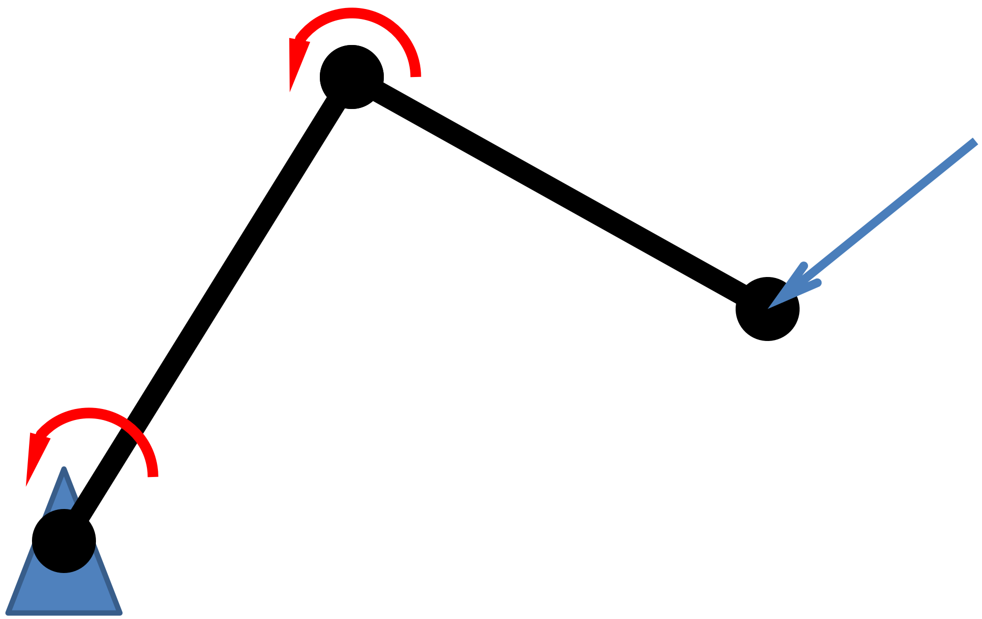


$$\mathbf{q}_{k+1} = \mathbf{q}_k + \mathbf{\Delta q} = \mathbf{q}_k + \mathbf{J}^{-1}(\mathbf{q}_k) \cdot \left(\alpha \cdot \mathbf{\Delta x} + \beta \cdot \mathbf{v}_m\right)$$

USING THE JACOBIAN FOR COMPUTING THE TORQUE

$$P_m = P_a \quad \begin{cases} P_m = \dot{\mathbf{q}}^T \mathbf{\Gamma} \\ P_a = \dot{\mathbf{x}}^T \mathbf{F} \end{cases}$$

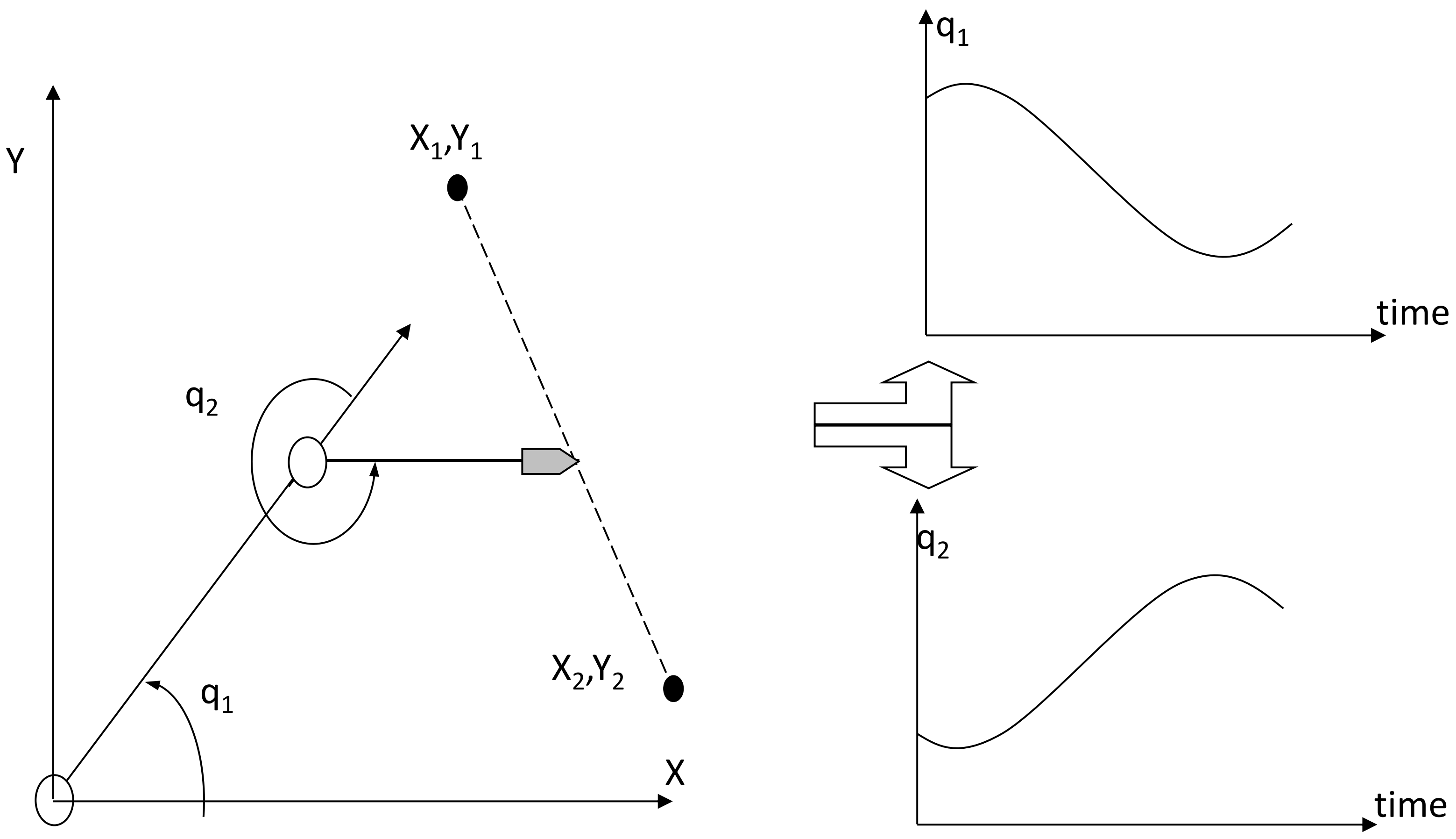
$$\mathbf{\Gamma} = J^T F$$



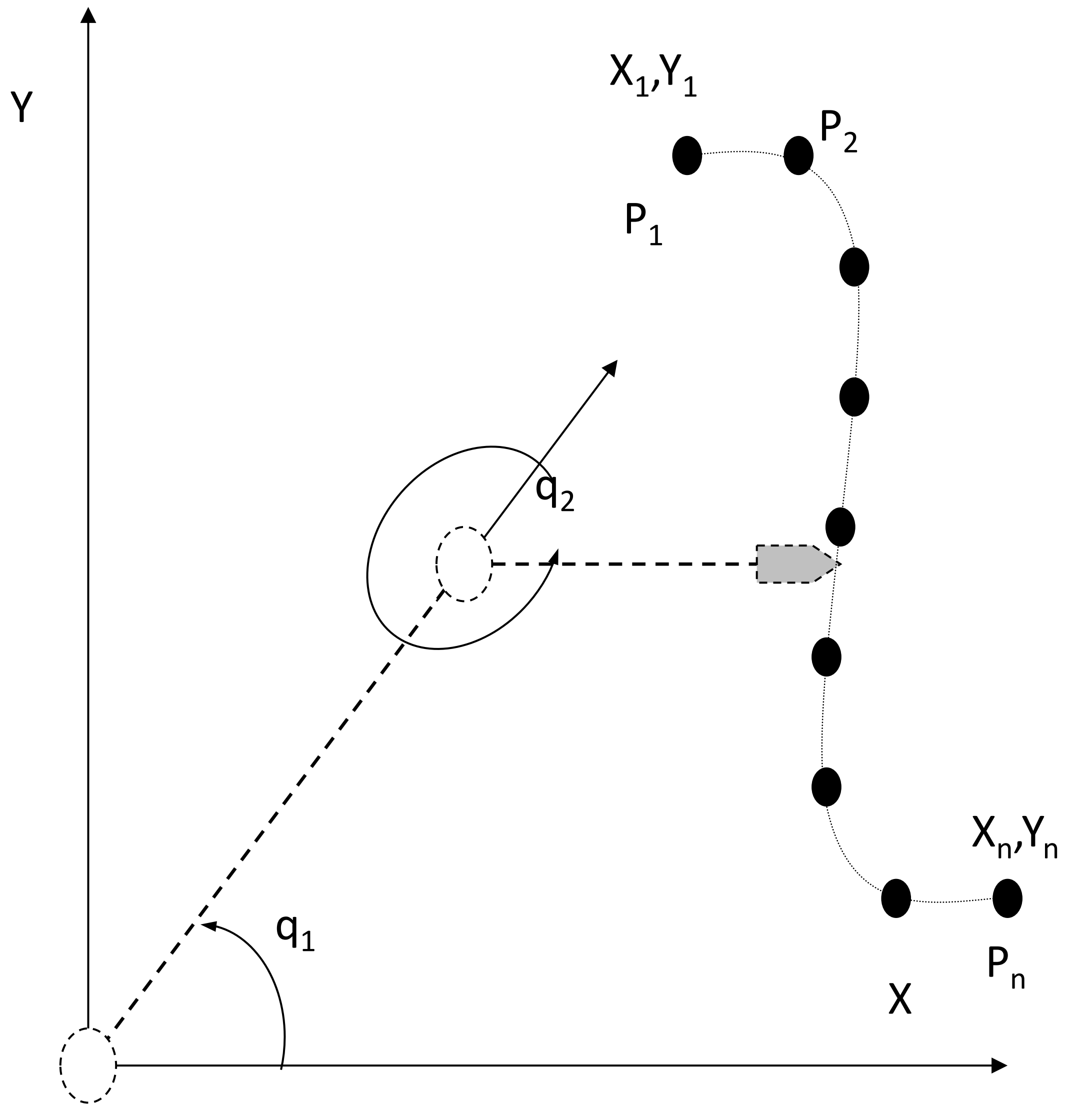
$$\mathbf{\Gamma} = \begin{bmatrix} \Gamma_1 \\ \dots \\ \Gamma_n \end{bmatrix}$$

$$\mathbf{F} = [F_x \quad F_y \quad F_z \quad M_x \quad M_y \quad M_z]^T$$

THE JOINT VARIABLES



Y
X_1,Y_1
q_2
q_1
X_2,Y_2
X
q_1
time
q_2
time



Y
X_1,Y_1
P_2
P_1
q_2
X_n,Y_n
q_1
X
P_n

$$q_i(t) = a_{i,0} + a_{i,1}t + a_{i,2}t^2 + a_{i,3}t^3$$

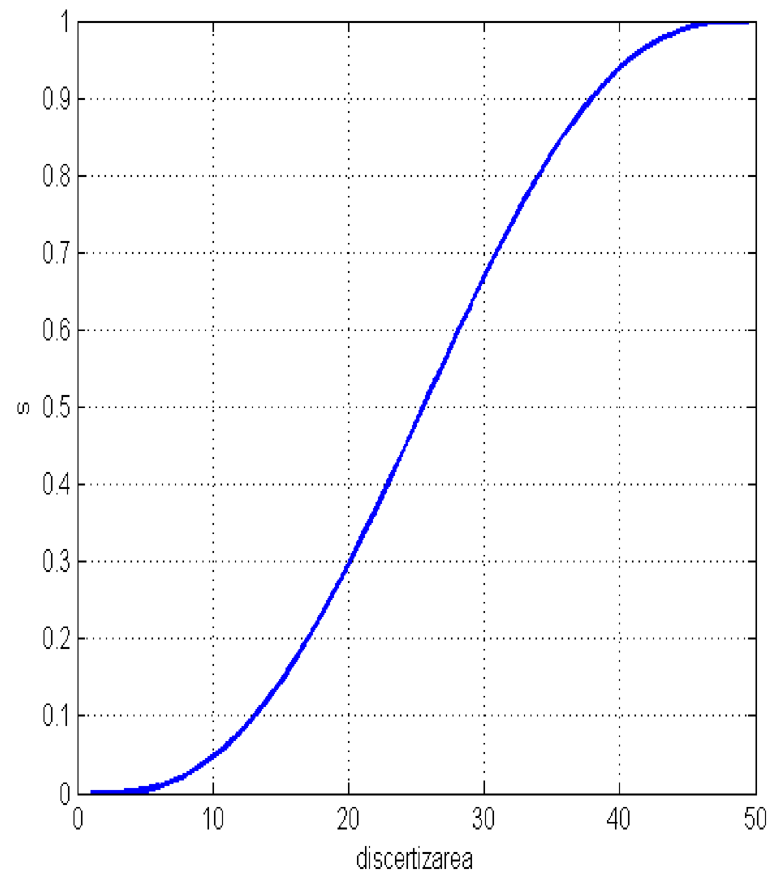
$$\begin{cases} q(0) = q_{i,1} \\ \quad\quad\;\; = a_{i,0} \\ \dot{q}(0) = \dot{q}_{i,1} = a_{i,1} \\ q(T) = q_{i,n} = a_{i,0} + a_{i,1}T + a_{i,2}T^2 + a_{i,3}T^3 \\ \dot{q}(T) = \dot{q}_{i,n} = a_{i,1} + 2a_{i,2}T + 3a_{i,3}T^2 \end{cases}$$

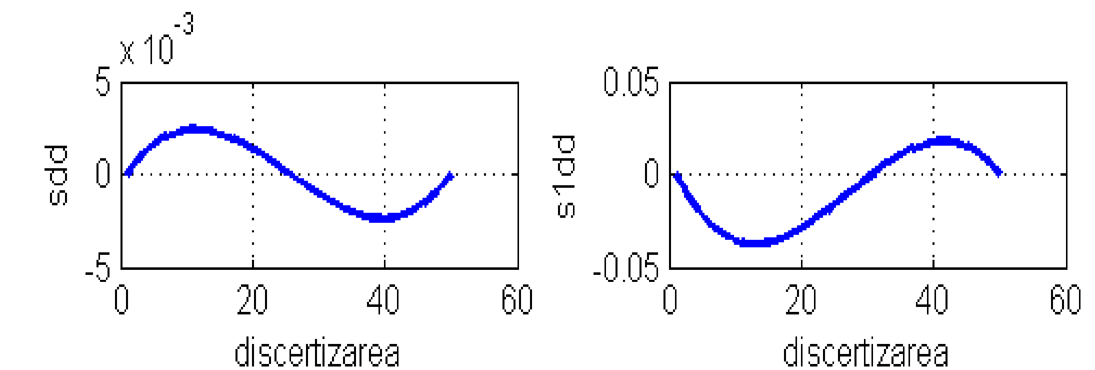
$$q_i(t) = a_{i,0} + a_{i,1}t + a_{i,2}t^2 + a_{i,3}t^3$$

$$\begin{cases} a_{i,0} = q_{i,1} \\ a_{i,1} = \dot{q}_{i,1} \\ a_{i,2} = \dfrac{3}{T^2}\left(q_{i,n} - q_{i,1}\right) - \dfrac{1}{T}\left(\dot{q}_{i,n} + 2\dot{q}_{i,1}\right) \\ a_{i,3} = \dfrac{2}{T^3}\left(q_{i,1} - q_{i,n}\right) + \dfrac{1}{T^2}\left(\dot{q}_{i,n} + \dot{q}_{i,1}\right) \end{cases}$$

$$t_j = t_{j-1} + \Delta_t \qquad j = \overline{1 \dots n}; t_0 = 0;\ t_n = T$$

$$\begin{cases} q_{i,j} = a_{i,0} + a_{i,1}t_j + a_{i,2}t_j^2 \\ \dot{q}_{i,j} = a_{i,1} + 2a_{i,2}t_j + 3 \\ \ddot{q}_{i,j} = 2a_{i,2} + 6a_{i,3}t \end{cases}$$




s
s1
sd
s1d
sdd
s1dd
x 10^-3
discertizarea

CONCUSIONS